1.0 RADAR BASICS

1.1 INTRODUCTION

According Wikipedia, Skolnik\(^1\) and other sources, the first attempt to detect targets using electromagnetic radiation took place in 1904 when Christian Hülsmeyer bounced waves off of a ship. During the 1920’s, several researchers, including R.C. Newhouse, G. Breit, M. A. Tuve, G. Marconi, L. S. Alder, and probably many others in the U.S. and other countries were obtaining patents on, and conducting experiments with, radar. Although these appear to be the first instances of the use of radar, the term radar was not applied to them. The name radar was coined in 1940 by the U.S. Navy as a contraction of RA dio Detection And Ranging. As with many other technology advancements, the early, significant advances in radar occurred during WWII. Since that time, radar technology has grown very rapidly, and is still advancing at a rapid pace. Radars see wide application in both commercial (airport radars, police radars, weather radar, etc.) and military arenas.

1.2 RADAR TYPES

Radars can use two types of signals:

- Pulsed, where the radar transmits a sequence of pulses of radio frequency (RF) energy
- CW (Continuous Wave), where the radar transmits a continuous signal.

The first example above was a CW radar whereas Breit & Tuve’s radar was a pulsed radar.

Since a CW radar transmits a continuous signal it requires the use of separate transmit and receive antennas because it is not (usually) possible to simultaneously receive while the radar is transmitting. Pulsed radars get around this problem by using what we might think of as time multiplexing. Specifically, during the time the pulse is transmitted the antenna is connected to the transmitter. After the transmit phase is completed the antenna is connected to the receiver. In the radar there is a device that is called a circulator that effectively performs this switching function. Pulsed radars are the most common type of radar because they only require one antenna.

The two basic types of radars are monostatic or bistatic radars.

- In a monostatic radar the transmitter and receiver, and associated antennas, are collocated. This is the most common type of radar because it is the most compact. If a monostatic radar is pulsed it will usually use the same antenna for transmit and receive. If a monostatic radar is CW it will usually have separate transmit and receive antennas with a shield between them.

- In a biastatic radar the transmitter and receiver are separated, often by very large distances (> 1Km). This is the type of radar that might be used in a

missile seeker. In this case, the transmitter is located on the ground, or in an aircraft, and the receiver is located in the missile.

As indicated earlier, the word radar is a contraction of **RAdio Detection And Ranging.** As implied by this contraction, radars are used to detect the presence of a target and to determine its location. The contraction implies that the quantity measured is range. While this is correct, modern radars are also used to measure range-rate and angle. It turns out that by measuring these parameters one can perform reasonably accurate calculations of the x-y-z location and velocity of a target. In some instances one can also form reasonable estimates of higher derivatives of x, y and z.

Radars operate in the radio frequency band of the energy spectrum between about 30 MHz (VHF) and 300 GHz (millimeter wave (mm)). A listing of frequency bands and associated frequencies are shown in Table 1-1. The bands shown in Table 1-1 are the U.S. IEEE waveband specifications. There is another set of waveband specifications called the European and U. S. ECM (electronic countermeasure) specification. It was popular for a while but has declined in popularity in the past 10 or so years. This table of waveband specifications can be found in the Wikipedia reference on Radio Spectrum.

<table>
<thead>
<tr>
<th>Band</th>
<th>Frequency Range</th>
<th>Origin of Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>VHF</td>
<td>30 – 300 MHz</td>
<td>Very High Frequency</td>
</tr>
<tr>
<td>UHF</td>
<td>300 – 1000 MHz</td>
<td>Ultra High Frequency</td>
</tr>
<tr>
<td>L</td>
<td>1 – 2 GHz</td>
<td>Long Wave</td>
</tr>
<tr>
<td>S</td>
<td>2 – 4 GHz</td>
<td>Short Wave</td>
</tr>
<tr>
<td>C</td>
<td>4 – 8 GHz</td>
<td>Compromise Between C and X Bands</td>
</tr>
<tr>
<td>X</td>
<td>8 – 12 GHz</td>
<td>Used for Fire Control radars in WWII. X stands for cross, as in cross hairs</td>
</tr>
<tr>
<td>Ku</td>
<td>12 – 18 GHz</td>
<td>Kurtz-Under</td>
</tr>
<tr>
<td>K</td>
<td>18 – 27 GHz</td>
<td>Kurtz – German for short wave</td>
</tr>
<tr>
<td>Ka</td>
<td>27 – 40 GHz</td>
<td>Kurtz-Above</td>
</tr>
<tr>
<td>V</td>
<td>40 – 75 GHz</td>
<td>Unknown</td>
</tr>
<tr>
<td>W</td>
<td>75 – 110 GHz</td>
<td>W follows V in the alphabet</td>
</tr>
<tr>
<td>mm</td>
<td>110 – 300 GHz</td>
<td>millimeter</td>
</tr>
</tbody>
</table>

Usually, but not always,

- Search radars operate at VHF to C band
- Track radars operate in X and Ku bands, and sometimes in K band
- Radars in missile active missile seekers operate in Ku and K band
- Instrumentation radars and short range radars sometimes operate in the Ka band and above.

Some notes on operating frequency considerations

- Low frequency radars require large antennas or have broader beams (broader distribution of energy in angle space – think of the beam of a flashlight). They are not usually associated with accurate angle measurement.
- Low frequency radars have limitations on range measurement accuracy because fine range measurement implies large instantaneous bandwidth of the transmit signal. This causes problems with transmitter and receiver design because the bandwidth could be a significant percentage of the transmit frequency.
- Doppler measurement is not accurate in low frequency radars because Doppler frequency is related to transmit frequency.
- High power is easier to generate at low frequencies because the devices can be larger and thus accommodate higher voltages and currents.
- For search, we want high power but we don’t necessarily need fine range or angle measurement. Thus, search radars tend to use lower frequencies.
- For track, we need fine range and angle measurement but we don’t necessarily need high power. Thus, track radars tend to use higher frequencies.
- The above also often leads to assigning the functions of search and track to different radars. However, modern radars tend to be “multi-function” and incorporate both functions in one radar. This can lead to trade offs in operating frequency, and in the search and track functions.

1.3 RANGE MEASUREMENT

The common way to measure range with a radar is to measure the time delay between transmission and reception of a pulse. This is illustrated in Figure 1-1. Since the RF energy travels at the speed of light, \( c \approx 3 \times 10^8 \) m/s, the time required for the transmit pulse to travel to a target at a range of \( R \) is

\[
\tau_{out} = \frac{R}{c}.
\]  \hspace{1cm} (1-1)

The time required for the pulse to return from the target back to the radar is

\[^2\text{We are assuming a monostatic radar in this discussion.}\]
Thus, the total round-trip delay between transmission and reception of the pulse is

\[ \tau_{\text{total}} = \tau_R + \tau_{\text{back}} = 2R/c. \]  

(1-3)

Since we can measure \( \tau_R \) in a radar, we can compute range by solving Equation (1-3) for \( R \); i.e.

\[ R = \frac{c\tau_R}{2}. \]  

(1-4)

**Rule of Thumb**

A rule-of-thumb concerning range measurement is as follows. Suppose \( \tau_R = \tau \mu s \). In other words, we express the time delay (which we often call range delay) in the units of micro-seconds. We can then write

\[ R = \frac{c}{2} \tau R = \frac{3 \times 10^8 \text{ m/s}}{2}\left(\tau \mu s \times 10^{-6} \text{ s/\mu s}\right) = 150\tau \text{ m.} \]  

(1-5)

In words, we say that we compute the range by multiplying the range delay, in \( \mu s \), by 150. Or that the range computation scaling factor is 150 meters per micro-second.

**1.4 AMBIGUOUS RANGE**

A problem with pulsed radars and range measurement is how to unambiguously determine the range to the target. This problem arises because of the fact that pulsed radars typically transmit a sequence of pulses. The issue is: where do we choose \( t = 0 \) in computing range delay? The common method is to choose it at the time of a transmit pulse. Thus, \( t = 0 \) is reset on each transmit pulse.
To define the problem, we consider the situation of Figure 1-2. In this figure the transmit pulses are spaced 400 µs apart. The target range is 90 Km, which means that the range delay to the target is

$$\tau_R = \frac{2R}{c} = \frac{2 \times 90 \times 10^3}{3 \times 10^8} \text{ s} = 60 \times 10^{-5} \text{ s} = 600 \mu s.$$  \hspace{1cm} (1-6)

This means that the return from pulse 1 would not be received until after pulse 2 is transmitted, the return from pulse 2 would not be received until after pulse 3 is transmitted, etc. Since all of the transmit pulses are the same and all received pulses are the same we have no way of associating received pulse 1 with transmit pulse 1. In fact, since we reset $t = 0$ on each transmit pulse, we will associate received pulse $k$ with transmit pulse $k + 1$. Further, we would measure the range delay as 200 µs. If we were to use this value of range delay to compute range we would get an apparent range of

$$R_A = 150 \tau = 150 \times 200 = 30,000 \text{ m or 30 Km}.$$  \hspace{1cm} (1-7)

Because of this we say that we have an ambiguity, or uncertainty, in measuring range.

![Figure 1-2 – Illustration of Ambiguous Range](image)

If the spacing between pulses is $\tau_{PRI}$ we say that the radar has an unambiguous range of

$$R_{amb} = \frac{c\tau_{PRI}}{2}. \hspace{1cm} (1-8)$$

This tells us that if the target range is less than $R_{amb}$ the radar can measure its range unambiguously. If the target range is greater than $R_{amb}$ we cannot measure its range unambiguously.

In the above notation, PRI stands for pulse repetition interval and is defined as the spacing between transmit pulses. A related term is pulse repetition frequency, or PRF. PRF is the reciprocal of PRI.

To avoid range ambiguities, radar designers typically choose the PRI to be greater than the range delay associated with the longest range targets of interest. They also limit the power to try to be sure that long range targets will not be detected by the radar.

©2011 M. C. Budge, Jr
Ambiguous range is a problem in search, but not track. In track, the radar tracking filters or algorithms have an idea of target range and can “look” in the proper place, even if the returns are ambiguous.

Another way to circumvent the ambiguous range problem is to use waveforms with multiple PRIs. That is, waveforms where the spacing between transmit pulses changes. An example of a multiple PRI, transmit waveform, and the appropriate received signal, is shown in Figure 1-3. In this figure, the spacings between the four transmit pulses are 400 µs, 300 µs and 350 µs. The target range delay is 600 µs, as in the previous example. It will be noted that the position of the number 1 received pulse, relative to the number 2 transmit pulse is 200 µs and the location of the number 2 received pulse, relative to the number 3 transmit pulse is 300 µs. The fact that the location of the received pulse relative to the most recently transmitted pulse is changing gives us the indication that the target range is ambiguous. We can use this fact to ignore the returns.

![Figure 1-3 – Staggered PRI Waveform and Ambiguous Range](image)

As an alternative, we could use the measured range delays in a range resolve algorithm to compute the true target range. Such an approach is used in pulsed Doppler radars because the PRIs used in these radars almost always result in ambiguous range operation.

Phased array radars, radars where the antenna beam is steered electronically rather than mechanically, often transmit a single pulse and then re-steer the beam to a different angular. In this type of radar, the concept of unambiguous range is not strictly defined since there is only one pulse. However, the term is still used in these radars. The unambiguous range is take to be the range delay associated with the time that the beam stays in one position, before moving to another position.

1.5 **USABLE RANGE AND INSTRUMENTED RANGE**

The above discussion of ambiguous range might lead one to conclude that a radar can detect (and track) targets at all ranges between 0 and $R_{amb}$. However, in practice, this is not the case. If a target were at a range of zero, a pulse that was received from it would arrive at the radar during the time that the sounding pulse was being transmitted. Since the receiver is off during this time, it could not process the pulse. Because of this, the minimum usable range is equal to

©2011 M. C. Budge, Jr
\( R_{\text{min}} = \frac{c \tau_p}{2} \)  

(1-9)

where \( \tau_p \) is the radar pulse width.

In a similar argument, if a pulse from a target arrives back at the receiver too close to the next transmit pulse, the receiver will again be off, and thus not able to process the received signal. This leads to the conclusion that the maximum usable range is

\[ R_{\text{max}} = \frac{c (\tau_{\text{PRI}} - \tau_p)}{2} . \]  

(1-10)

That is, the maximum usable range extends to one pulse width before the next transmit pulse. There are occasions where these bounds can be exceeded a little, however, this is not often done because it can lead to processing difficulties.

Although, \( R_{\text{min}} \) and \( R_{\text{max}} \) define the maximum usable range extent of a radar, most radars operate over a shorter range interval that is termed the *instrumented range*. The instrumented range is set by system requirements and allows factors such as display limits, time to allow for circuit transients, time to allow for radar calibration, and the like.

### 1.6 RANGE-RATE MEASUREMENT (DOPPLER)

In addition to range measurement, radars can also be used to measure the rate-of-change of range, or *range-rate*. This is done by measuring the *Doppler frequency*; that is, the frequency difference between the transmitted and received signals.\(^3\) To examine this further we consider the geometry of *Figure 1-4*. In this figure, the aircraft is moving in a straight line at a velocity of \( v \). As a result, the range to the target is continually changing. Indeed, over a differential time of \( dt \) the range will change by an amount \( dR \), from \( R \) to \( R + dR \). The resulting rate-of-change of range, or range-rate, will thus be

\[ \dot{R} = \frac{dR}{dt} . \]  

(1-11)

We note that in *Figure 1-4* the range-rate will be negative because the range is decreasing from time \( t \) to time \( t + dt \).

---

\(^3\) This is the same Doppler effect you studied in physics.
We note that, in general, $\dot{R} \neq v$. Equality will hold only if the target is flying along a radial path relative to the radar.

As a digression, we want to relate range-rate to target position and velocity in a Cartesian coordinate system. Suppose the target position and velocity state vector is given by

$$X = [x \ y \ z \ \dot{x} \ \dot{y} \ \dot{z}]^T$$

where the superscript $T$ denotes the transpose. We can write the target range as

$$R = \sqrt{x^2 + y^2 + z^2}$$

and the range-rate as

$$\dot{R} = \frac{dx}{dt} = \frac{x\ddot{x} + y\ddot{y} + z\ddot{z}}{R}.$$  \hspace{1cm} (1-14)

Now let’s return to the problem of showing how to measure range-rate with a radar. To start we need to think about the nature of the transmit pulse. The simplest form of a transmit pulse is a snippet of a sinusoid where the frequency of the sinusoid is the operating frequency of the radar (e.g. $100 \times 10^6$ Hz, or 100 MHz for a radar operating at VHF, $10^9$ Hz, or 1 GHz (giga-hertz) for a radar operating at L-band or $10 \times 10^9$ Hz, or 10 GHz, for a radar operating at X-band). We normally term the operating frequency the carrier frequency of the radar and denote it as $f_c$ or $f_o$. An example of a transmit pulse is shown in Figure 1-5. This figure is not to scale in that it shows only 5 cycles of the carrier over the duration of the pulse. If we consider an X-band radar and a pulse duration, or pulse width, of $\tau_p = 1 \mu s$, there will be 10,000 cycles of the carrier over the duration of the pulse.
We can mathematically represent the transmit pulse as

\[ v_t(t) = \text{rect} \left( \frac{t}{\tau_p} \right) \cos \left( 2\pi f_c t \right) \]  \hspace{1cm} (1-15)

where

\[ \text{rect} \left( \frac{t}{\tau_p} \right) = \begin{cases} 1 & 0 \leq t < \tau_p \\ 0 & \text{elsewhere} \end{cases} \]  \hspace{1cm} (1-16)

The signal received by the radar is, ideally, an attenuated, delayed version of the transmit signal, i.e.

\[ v_r(t) = A v_t(t - \tau_R) \]  \hspace{1cm} (1-17)

where the amplitude scaling factor, \( A \), comes from the radar range equation, and the delay, \( \tau_R \), is the range delay discussed in the previous section. To compute Doppler frequency we must acknowledge that the range delay in Equation (1-17) is a function of time, \( t \), and write

\[ \tau_R(t) = \frac{2R(t)}{c} \]  \hspace{1cm} (1-18)

If we substitute Equation (1-18) into Equation (1-17) we get

\[ v_r(t) = A \text{rect} \left( \frac{t - \tau_R(t)}{\tau_p} \right) \cos \left( 2\pi f_c (t - \tau_R(t)) \right) = A \text{rect} \left( \frac{t - \tau_R(t)}{\tau_p} \right) \cos(\phi(t)) \]  \hspace{1cm} (1-19)

The information we seek is in the argument of the cosine term.\(^4\) This means that we want to examine

\(^4\) The impact of the time-varying range delay on the rect(•) function is that the returned pulse will be slightly shorter or longer than the transmit pulse. However, the difference in pulse widths is usually very small and can be neglected.
\[ \phi(t) = 2\pi f_c (t - \tau_R(t)) = 2\pi f_c \left( t - 2R(t)/c \right). \]  

(1-20)

We start by expanding \( R(t) \) into its Taylor series to get

\[ R(t) = R(0) + \dot{R}(0)t + \ddot{R}(0)t^2/2! + \dddot{R}(0)t^3/3! + \cdots \]

\[ = R + \dot{R}t + g(t^2, t^3, \cdots) \]  

(1-21)

If we substitute Equation (1-21) into Equation (1-20) we get

\[ \phi(t) = 2\pi f_c t - 2\pi f_c \left( R + \dot{R}t + g(t^2, t^3, \cdots) \right) \]

\[ = 2\pi f_c t - 2\pi f_c \left( 2R/c \right) - 2\pi f_c \left( 2\dot{R}/c \right)t - 2\pi f_c g(t^2, t^3, \cdots) \]  

(1-22)

or

\[ \phi(t) = 2\pi f_c t + \phi_r + 2\pi f_d t + \varphi'\left(t^2, t^3, \cdots\right). \]  

(1-23)

In Equation (1-23)

- \( \phi_r = -2\pi f_c \left( 2R/c \right) \) is the phase shift due to range delay and is of little use in practical radars.
- \( g'\left(t^2, t^3, \cdots\right) = -4\pi f_c g(t^2, t^3, \cdots)/c \) is a non-linear phase term that is usually ignored (until it causes problems in advanced signal processors)
- \( f_d = -f_c \left( 2\dot{R}/c \right) \) is the Doppler frequency of the target.

We can make use of the fact that the radar wavelength, \( \lambda \), is given by

\[ \lambda = c/f_c \]  

(1-24)

to rewrite \( f_d \) in its more standard form of

\[ f_d = -2\dot{R}/\lambda. \]  

(1-25)

If we drop the \( g'\left(t^2, t^3, \cdots\right) \) term and substitute Equation (1-23) into Equation (1-19), without the \( g'\left(t^2, t^3, \cdots\right) \) term, we get

\[ v_R(t) = A \text{rect} \left[ \frac{t - \tau_R}{\tau_p} \right] \cos \left( 2\pi \left( f_c + f_d \right)t + \phi_R \right) \]  

(1-26)

where we have used \( \tau_R = 2R(0)/c \). In Equation (1-26) we note that the frequency of the returned signal is \( f_c + f_d \) instead of simply \( f_c \). Thus, if we compare the frequency of the transmit signal to the frequency of the received signal we can determine the Doppler frequency, \( f_d \). Once we have \( f_d \) we can compute the range-rate from Equation (1-25).
In practice, measuring Doppler frequency is not as easy as implied above. The problem lies in the relative magnitudes of \( f_d \) and \( f_c \). To see this we consider a typical example. We start by assuming that we have a target that is traveling at about Mach 0.5, or about 150 m/s. For now, we assume that the target is flying directly toward the radar so that \( \dot{R} = -v = -150 \text{ m/s} \). We further assume that the radar is operating at X-band with a specific carrier frequency of \( f_c = 10 \text{ GHz} \). With this we get

\[
\lambda = \frac{c}{f_c} = \frac{3 \times 10^8}{10 \times 10^9} = 0.03 \text{ m} \tag{1-27}
\]

and

\[
f_d = -\frac{2\dot{R}}{\lambda} = -\frac{2(-150)}{0.03} = 10^4 \text{ Hz} = 10 \text{ KHz} \tag{1-28}
\]

If we compare \( f_c \) to \( f_d \) we note that \( f_d \) is a million times smaller than \( f_c \).

Although the measurement of Doppler frequency is not easy, it is doable. To measure Doppler frequency the transmit pulse must be very long (in the order of ms rather than µs) or the Doppler measurement must be based on processing several pulses. We will defer a detailed of how Doppler frequency is measured until a later date.

### 1.7 DECIBELS

To close out this section we want to discuss a measurement convention commonly used in radar analyses – the decibel. The concept of the decibel was originally formulated by engineers at Bell Telephone Laboratories as a measure of losses over given distances of telephone cable. By definition, a decibel representation of a quantity is equal to 10 times the logarithm to the base 10 of that quantity. As implied by its name, a decibel is 1/10 of a bel, which is the logarithm of a quantity. The name was coined by engineers at Bell Labs in honor of Alexander Graham Bell.

The decibel, abbreviated dB, is useful in radar applications because of the large range of numbers encountered in radar analyses. dB by itself is taken to mean \(10\log(\text{power ratio})\). Thus, when we represent signal-to-noise ratio in dB we compute

\[
\text{SNR}\big|_{\text{dB}} = 10\log \left( \frac{P_s}{P_N} \right) \tag{1-29}
\]

where \( P_s \) is the signal power (in watts, milliwatts, kilowatts, etc.) and \( P_N \) is the noise power (in the same units as \( P_s \)).

Equation (1-29) is the “standard” use of dB; the one originally defined by Bell Labs. Since that time, radar, electronics and communications analysts have expanded the use of dB to include many other forms. Some that we will encounter in this course include

- dBW – power level relative to 1 watt – \(10\log(P)\) where \( P \) is power in watts
- dBm – power level relative to 1 milliwatt – $10\log(P/0.001) = 30 + 10\log(P)$.
- dBsm – area in square meters relative to 1 square meter – $10\log(A)$ where $A$ is area in square meters. We use this to represent the radar cross-section of a target.
- dBi – Antenna gain relative to the gain of an isotropic antenna – $10\log(G)$ where $G$ is the antenna gain in w/w (watt/watt). The gain of an isotropic antenna is taken to be 1 w/w.