2.0 RADAR RANGE EQUATION

2.1 INTRODUCTION

One of the simpler equations of radar theory is the radar range equation. Although it is one of the simpler equations, ironically, it is an equation that few radar analysts understand and many radar analysts misuse. The problem lies not with the equation itself but with the various terms that make-up the equation. It is my belief that if one really understands the radar range equation one will have a very solid foundation in the fundamentals of radar theory. Because of the difficulties associated with using and understanding the radar range equation we will devote considerable class time to it and to the things it impacts, like detection theory, matched filters and the ambiguity function.

2.2 BASIC RADAR RANGE EQUATION

One form of the basic radar range equation is

$$SNR = \frac{P_S}{P_N} = \frac{P_T G_T G_R \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 B F_n L} \tag{2-1}$$

where

- *SNR* is termed the signal-to-noise ratio and has the units of watts/watt, or w/w.
- P_S is the signal power at some point in the radar receiver – usually at the output of the matched filter or the signal processor. It has the units of watts (w).
- P_N is the noise power at the same point that P_S is specified and has the units of watts.
- P_T is termed the *peak transmit power* and is the average power when the radar is transmitting a signal. P_T can be specified at the output of the transmitter or at some other point like the output of the antenna feed. It has the units of watts
- G_T is the directive gain of the transmit antenna and has the units of w/w.
- G_R is the directive gain of the receive antenna and has the units of w/w. Usually, $G_T = G_R$ for monostatic radars.
- λ is the radar wavelength (see Equation (1-25) of the Radar Basics chapter) and has the units of meters (m).
- σ is the target *radar cross-section or RCS* and has the units of square meters or m^2.
- R is the range from the radar to the target and has the units of meters.

- k is Boltzman's constant and is equal to $1.38\times10^{-23}\ \text{w}/\left(\text{Hz}\ ^\circ\text{K}\right)$.
- T_0 denotes a reference temperature in degrees Kelvin $\left(^\circ\text{K}\right)$. We take $T_0 = 290\ ^\circ\text{K}$ and usually use the approximation $kT_0 = 4\times10^{-21}$ w/Hz.
- B is the ***effective*** noise bandwidth of the radar and has the units of Hz. I emphasized the word effective because this point is extremely important and often misunderstood and misused by radar analysts.
- F_n is the radar *noise figure* and is dimensionless, or has the units of w/w.
- L is a term included to account for all losses that must be considered when using the radar range equation. It accounts for losses that apply to the signal and not the noise. L has the units of w/w. L accounts for a multitude of factors that degrade radar performance. These factors include those related to the radar itself, the environment in which the radar operates, the radar operators and, often, the ignorance of the radar analyst.

We will spend the next several pages deriving the radar range equation and attempting to carefully explain its various terms and their origins. We will start by deriving P_S, or the signal power component and follow this by a derivation P_N, or the noise component.

2.2.1 Derivation of P_S

2.2.1.1 The Transmitter

We will start at the transmitter output and go through the waveguide and antenna and out into space, see *Figure 2-1*. For now, we assume that the radar is in free space. We can account for the effects of the atmosphere at a later date. We assume that the transmitter generates a single, rectangular pulse (a standard assumption) at some carrier frequency, f_c. A sketch of the *pulse* (the terminology we use) is contained in *Figure 2-2*. The average power in the signal *over the duration of the pulse* is termed the *peak* transmit power and is denoted as P_T. The reason we term this power the peak transmit power is that we will later want to consider the transmit power averaged over many pulses.

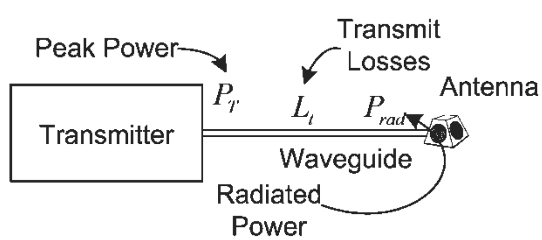

Figure 2-1 – Transmit Section of a Radar

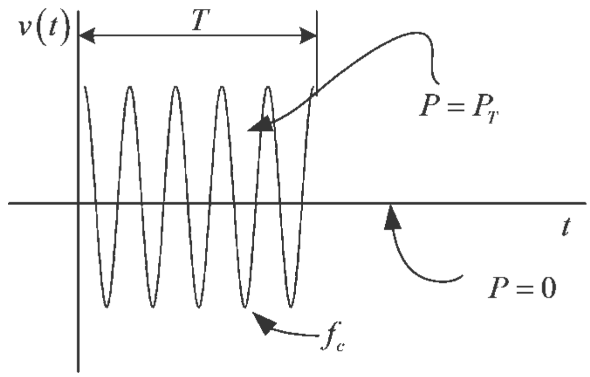

Figure 2-2 – Depiction of a Transmit Pulse

The waveguide of *Figure 2-1* carries the signal from the transmitter to the antenna feed input. The only feature of the waveguide that is of interest in the radar range equation is the fact that it is a lossy device that attenuates the signal. Although we only refer to the "waveguide" here, in a practical radar there are several devices between the transmitter and antenna feed. We lump all of these into a conceptual waveguide.

Since the waveguide is a lossy device we characterize it in terms of its loss which we denote as L_t and term *transmit loss*. Since L_t is a loss it is greater than unity. With this, the power at the input to the feed is

$$P_{rad} = \frac{P_T}{L_t} \text{ w} \tag{2-2}$$

and is termed the *radiated power*.

We assume that the feed and the antenna are ideal and thus introduce no additional losses to the radiated power. In actuality, the antenna assembly (antenna and feed) will have losses associated with it. These losses are incorporated in L_t. Because of the above assumption, the power radiated into space is P_{rad}.

2.2.1.2 <u>The Antenna</u>

The purpose of the radar antenna is to concentrate, or focus, the radiated power in a small angular sector of space. In this fashion, the radar antenna works much as the reflector in a flashlight. As with a flashlight, a radar antenna doesn't perfectly focus the beam. However, for now we will assume it does. Later, we will account for the fact that the focusing isn't perfect by a scaling term.

With the above, we assume that all of the radiated power is concentrated in an area, A_{beam}, as indicated in *Figure 2-3*. Therefore, the power density over A_{beam} is

$$S_R = \frac{P_{rad}}{A_{beam}} = \frac{P_T / L_t}{A_{beam}} \text{ w/m}^2 . \tag{2-3}$$

To carry Equation (2-3) to the next step we need an equation for A_{beam}. Given that the lengths of the major and minor axes of the ellipse in *Figure 2-3* are $R\theta_A$ and $R\theta_B$, we can write the area of the ellipse as

$$A_{ellipse} = \frac{\pi}{4} R^2 \theta_A \theta_B \text{ m}^2. \quad (2\text{-}4)$$

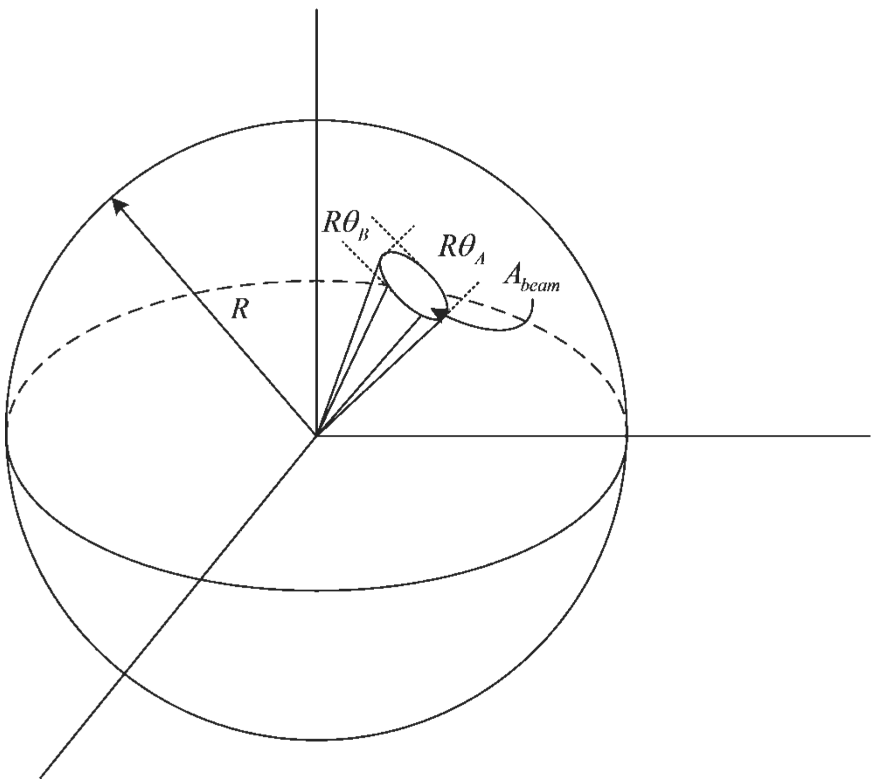

Figure 2-3 – Radiation Sphere with Antenna Beam

We next recognize that the power is not uniformly distributed across $A_{ellipse}$ and that some of the power will "spill" out of the area $A_{ellipse}$. We account for this by replacing $\pi/4$ with a constant K_A, which will be less than $\pi/4$. We will further discuss K_A shortly. With this we write A_{beam} as

$$A_{beam} = K_A R^2 \theta_A \theta_B \text{ m}^2. \quad (2\text{-}5)$$

If we substitute Equation (2-5) into Equation (2-3) we get

$$S_R = \frac{P_T / L_t}{K_A R^2 \theta_A \theta_B} \text{ w/m}^2. \quad (2\text{-}6)$$

At this point we define a term, G_T, that we call the transmit *antenna gain* as

$$G_T = \frac{4\pi}{K_A \theta_A \theta_B} \text{ w/w} \tag{2-7}$$

and use it to rewrite Equation (2-6) as

$$S_R = \frac{G_T P_T}{4\pi R^2 L_t} \text{ w/m}^2 . \tag{2-8}$$

As it is used here, G_T, is termed *directive gain*.[1] With this form of the antenna gain, it is assumed that if there are losses in the feed or the antenna itself, these losses will be included as separate loss terms in the radar range equation (losses will be discussed in a later section). Some analysts include the feed and antenna losses in the transmit antenna gain and term the result the *power gain* of the antenna. We will not do that here since it can lead to confusion when using Equation (2-7) and a form of directive gain to be presented shortly.

2.2.1.3 Effective Radiated Power

We now want to discuss a quantity termed *effective radiated power*. To do so we ask the question: What power would we need at the feed of an isotropic radiator to get a power density of S_R at all points on a sphere of radius R? An isotropic radiator is an antenna that does not focus energy; instead it distributes power uniformly over the surface of a sphere centered on the antenna. We can think of an isotropic radiator as a point source radiator. We note that an isotropic radiator cannot exist in the "real world". However, it is a mathematical and conceptual concept that we often use in radar theory, like the impulse function is in mathematical theory.

If we denote the effective radiated power as P_{eff} and realize that the surface area of a sphere of radius R is $4\pi R^2$ we can write the power density on the surface of the sphere as

$$S_R = \frac{P_{eff}}{4\pi R^2} \text{ w/m}^2 . \tag{2-9}$$

If we equate Equation (2-8) and Equation (2-9) we obtain

$$P_{eff} = \frac{P_T G_T}{L_t L_{ant}} \text{ w } = ERP \tag{2-10}$$

as the effective radiated power, or ERP. In this equation, the losses in the feed and/or antenna are included as the L_{ant} term.

Some radar analysts think that the power at the output of a radar antenna is the ERP. **IT IS NOT**. The power at the output of the antenna is $P_T / L_t L_{ant}$. All the antenna does if focus this power over a relatively small angular sector.

[1] Skolnik, M. I., Introduction to Radar Systems, Third Edition, 2001, McGraw-Hill, New York, N.Y.

Another note is that the development above makes the tacit assumption that the antenna is pointed exactly at the target. If the antenna is not pointed at the target, G_T must be modified to account for this. We do this by means of an *antenna pattern* which is a function that gives the value of G_T at all possible angles of the target relative to where the antenna is pointing.

2.2.1.4 Antenna Gain

We next want to address the factor K_A in Equation (2-7). As was indicated, K_A accounts for the properties of the antenna. Specifically:

- It accounts for the fact that the power is not uniformly distributed over the ellipse.
- It accounts for the fact that not all of the power is concentrated in the antenna beam, the ellipse. Some of it will "spill" out of the beam into what we term the *antenna sidelobes*.

A good value for K_A is 1.65. With this we can write the antenna gain as

$$G_T = \frac{4\pi}{1.65\theta_A\theta_B} \text{ w/w} . \tag{2-11}$$

In Equation (2-11) the quantities θ_A and θ_B are termed the *antenna beamwidths* and have the units of radians. In many applications, θ_A and θ_B are specified in degrees. In this case we write the gain as

$$G_T = \frac{25{,}000}{\theta_A^\circ\theta_B^\circ} \text{ w/w} \tag{2-12}$$

where the two beamwidths in the denominator are in degrees.

To visualize the concept of beamwidth we consider *Figure 4* which is a plot of $G_T(\theta,\phi)$ vs. θ for $\phi=0$. The expression $G_T(\theta,\phi)$ is a means of saying that the antenna gain is a function of where the target is located relative to where the antenna is pointing. With some thought, you will realize that two angles are needed to specify any point on the sphere discussed earlier.

The unit of measure on the vertical axis is dBi, or decibels relative to isotropic (see Chapter 1) and is the common unit of measure for G_T in radar applications. We define the beamwidth of an antenna as the distance between the *3-dB points*[2] of *Figure 4*. The 3-dB points are the angles where $G_T(\theta,\phi)$ is 3 dB below its maximum value. As a side note, the maximum value of $G_T(\theta,\phi)$ is the antenna gain, or G_T. With this we find that the antenna represented in *Figure 4* has a beamwidth of 2 degrees. We might call

[2] The concept of 3-dB points should be familiar from control and signal processing theory in that it is the standard measure used to characterize bandwidth.

this θ_A°. Suppose we were to plot $G_T\left(\theta,\phi\right)$ vs. θ for $\phi = 90^\circ$. and find distance between the 3-dB points was 2.5 degrees. We would then say that the beamwidth was 2.5 degrees in this direction. We would then call this θ_B°. We would compute the antenna gain as

$$G_T = \frac{25,000}{2\times 2.5} = 5000 \text{ w/w or 37 dBi}. \qquad (2\text{-}13)$$

In the future, we will drop the notation dBi and use dB.

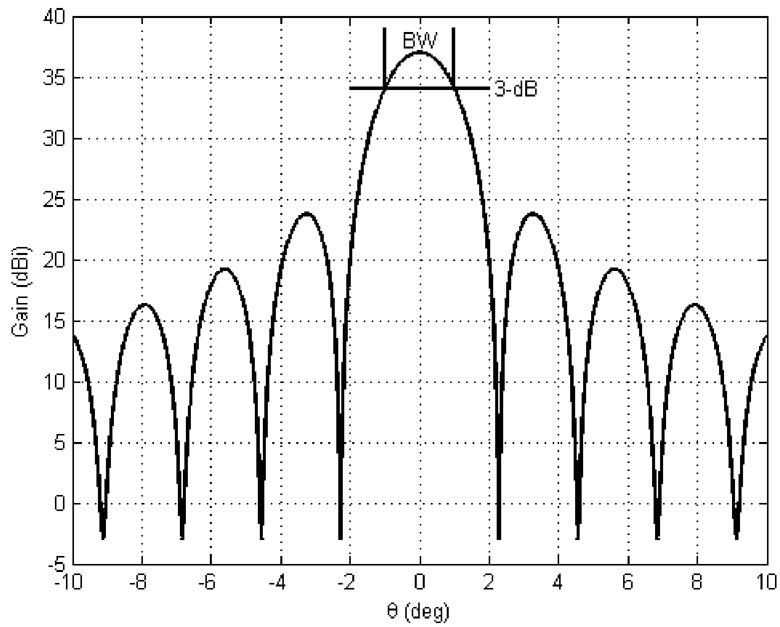

Figure 2-4 – Sample Antenna Pattern

The humps in *Figure 2-4* on either side of the central hump are the sidelobe of the antenna that were mentioned earlier.

2.2.1.5 <u>The Target and Radar Cross-Section</u>

Now back to our derivation. Thus far we have an equation for S_R, the power density in the location of the target. As the electromagnetic wave passes by the target, some of the power in it is captured by the target and re-radiated back toward the radar. The process of capturing and re-radiating the power is very complicated and the subject of much research. We will discuss it further in later sections. For now we simplify the process by using the concept of *radar cross-section or RCS*. We note that S_R has the units of w/m^2. Therefore, if we were to multiply S_R by an area we would convert it to a power. This is what we do with RCS, which we denote by σ and ascribe the units of m^2, or dBsm if we convert it to dB units. With this we say that the power captured and re-radiated by the target is

$$P_{tgt} = \sigma S_R \text{ w}. \tag{2-14}$$

We further make the idealized assumption that the target acts as an isotropic antenna and radiates P_{tgt} uniformly in all directions. In this sense, P_{tgt} is interpreted as an *effective* radiated power In fact, the target is much like an actual antenna and radiates the power with different amplitudes in different directions. Again, this process is very complicated and beyond the scope of this course.

To get an ideal of the variation of P_{tgt} refer to Figure 2.15 of the Skolnik reference of footnote 1. This figure indicates that P_{tgt} can vary by about 25 dB depending upon the orientation of the aircraft relative to the radar.

Given the above assumption that the power radiated by the target is P_{tgt} and that it acts as an isotropic radiator, the power density at the radar is

$$S_{rec} = \frac{P_{tgt}}{4\pi R^2}. \tag{2-15}$$

Or, substituting Equation (2-8) into Equation (2-14), and the result into Equation (2-15),

$$S_{rec} = \frac{P_T G_T \sigma}{(4\pi)^2 R^4 L_t}.^{3} \tag{2-16}$$

2.2.1.6 Antenna Again

As the electromagnetic wave from the target passes the radar, the radar antenna captures part of it and sends it to the radar receiver. If we follow the logic we used for the target, we can say that the power at the output of the antenna feed is

$$P_{ant} = S_{rec} A_e \tag{2-17}$$

where A_e is the *effective* area of the antenna; it is an area measure that describes the ability of the antenna to capture the returned electromagnetic energy and convert it into usable power. A more common term for A_e is *effective aperture* of the antenna. According to the dictionary, aperture means opening or orifice. Thus, in this context, we can think of the antenna as an orifice that funnels energy into the radar.

It turns out that the effective aperture is related to the physical area of the antenna. That is

$$A_e = \rho A_{ant} \tag{2-18}$$

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[3] We have included the extra loss term, L_{ant}, discussed earlier. Thus we are assuming the antenna and feed are lossless. We will correct this potential oversight when we consider loss terms.

where A_{ant} is the area of the antenna projected onto a plane placed directly in front of the antenna. We make this clarification of area because we don't want to confuse it with the actual surface area of the antenna. If the antenna is a parabola of revolution (a paraboloid), a common type of antenna, the actual area of the antenna would be the area of the paraboloidal surface of the antenna.

If we substitute Equation (2-16) into Equation (2-17) we get

$$P_{ant} = \frac{P_T G_T \sigma A_e}{(4\pi)^2 R^4 L_t}. \tag{2-19}$$

2.2.1.7 Antenna Gain Again

It turns out that Equation (2-19) is not usually very easy to use because of the A_e term. A more convenient method of characterizing the antenna would be through the use of its gain, as we did on transmit. According to antenna theory, we can relate antenna gain to effective aperture by the equation

$$G_R = \frac{4\pi A_e}{\lambda^2}. \tag{2-20}$$

If we substitute Equation (2-20) into Equation (2-19) we get

$$P_{ant} = \frac{P_T G_t G_R \lambda^2 \sigma}{(4\pi)^3 R^4 L_t}. \tag{2-21}$$

2.2.1.8 Losses

As a final step in this part of the development, we need to account for losses that we have ignored thus far. Two of these are the antenna and feed loss that we discussed earlier, and a similar loss for the antenna and feed on receive. It turns out that there are many more losses that we will need to account for. For now we will lump all of these losses with L_t and denote them by L. With this we say that the signal power in the radar is given by

$$P_S = \frac{P_T G_t G_R \lambda^2 \sigma}{(4\pi)^3 R^4 L} \tag{2-22}$$

which is P_{ant} with the additional losses added in.

In the above paragraph we said that P_S is the signal power "in the radar". However, we didn't say where in the radar. We will save this discussion until later. For now, we want to turn our attention to the noise term, P_N.

2.2.2 Derivation of P_N

All radars, as with all electronic equipment, must operate in the presence of noise. In electronic devices the main source of noise is termed *thermal noise* and is due to agitation of electrons caused by heat. The heat can be caused by the environment (the sun, the earth, the room, humans, etc.) and by the electronic equipment itself. In most radars the predominant source of heat is the electronic equipment.

2.2.2.1 Noise Power Spectral Density

Eventually, we want to characterize noise in terms of its power at some point in the receiver. However, to derive this power, and provide a means of characterizing the effects of the receiver electronics, we will use the approach commonly used in radar and communication theory. With this approach, we start by assuming that the noise in the radar, before we need to represent it as power, is white. Because of this, we start by characterizing the noise in terms of its *power spectral density*, or *energy*, which are the same in this context. (We can't use power because white noise has infinite power.) We define the noise power spectral density in the radar by the equation

$$N_0 = kT_{eff} \text{ w/Hz or w-s or joule} \tag{2-23}$$

where $k = 1.38054 \times 10^{-23}$ w/$\left(\text{Hz} \times {}^\circ\text{K}\right)$ is Boltzman's constant. T_{eff} is the *effective noise temperature* of the radar in degrees Kelvin (ºK). It turns out that T_{eff} is not an actual temperature. Rather, it is a temperature quantity that we use to compute the proper noise power spectral density in the radar. Although this may be confusing at present (it says that we need to know N_0 to compute T_{eff} which we need to compute N_0 !), it will hopefully become clearer when we undertake a more detailed discussion of noise.

2.2.2.2 Noise Figure

As an alternate formulation we also write

$$T_{eff} = F_n T_0 \tag{2-24}$$

where F_n is termed the *noise figure* of the radar and T_0 is a reference temperature normally referred to as "room temperature". In fact, $T_0 = 290\ {}^\circ\text{K}$ or 16.84 ºC (0 ºC = 273.16 ºK) or about 62 ºF which, by some standards, may be room temperature. With the above we get

$$N_0 = kT_0 F_n . \tag{2-25}$$

It is interesting to note that $kT_0 = 4\times10^{-21}$ w/Hz, which makes one think that the value of $T_0 = 290\ ^\circ\mathrm{K}$ was chosen to make kT_0 a "nice" number, and not because it is room temperature.

2.2.2.3 Effective Noise Bandwidth

Since N_0 has the units of w/Hz, we need to multiply it by a frequency term to convert it to a power. In fact, we use this idea to write the noise power in the radar as

$$P_N = kT_0F_nB \tag{2-26}$$

where we term B the *effective noise bandwidth* of the radar. We want to emphasize the term ***effective***. In fact, B may not be the actual bandwidth of any component of the radar. It turns out that if the radar transmits a single, rectangular pulse (as in *Figure 2-2*), and if the receiver employs a filter that is *matched* to the transmit pulse, and we are trying to represent the power at the output of this *matched filter* then, in terms of the radar range equation, B is the bandwidth of the matched filter. It will be left as a homework problem to determine if B is the 3-dB bandwidth of the matched filter. It will be noted that I placed a lot of caveats on our ability to tie B to a specific bandwidth. I did this to emphasize that we must be very careful in how we define the noise in a radar. A very common mistake in the use of the radar range equation is to use the transmit waveform bandwidth for B. For modern, pulse-compression radars this is ***incorrect***!

If we combine Equation (2-26) and Equation (2-22) with the relation $SNR = P_S/P_N$ we get Equation (2-1) or

$$SNR = \frac{P_S}{P_N} = \frac{P_TG_TG_R\lambda^2\sigma}{(4\pi)^3 R^4kT_0BF_nL}\ \text{w/w} \tag{2-27}$$

What we have not done in Equation (2-1) and Equation (2-27) is state where in the radar we are characterizing the SNR. We will do this after we discuss some other topics. For now, we want to develop an alternate formulation for SNR which, with one relation, will take the same form as (2-1) and (2-27).

2.3 AN ENERGY APPROACH TO SNR

In this approach to SNR we define the SNR as the ratio of the signal energy to the noise energy (which, you will recall, is the power spectral density). Recall that Equation (2-22) is the signal power in the radar (again, we won't say where yet). We further assume that the shape of the originally transmitted pulse is preserved. This means that at the point we measure the signal it has a power (peak power) of P_S for a duration of τ_p, the pulse width, and zero at all other times.[4] This means that the energy in the signal is

$$E_S = P_S \tau_p \text{ w-s or joule}. \tag{2-28}$$

The energy in the noise is given by Equation (2-25) and is

$$E_N = N_0 = kT_0F_n \text{ joule}. \tag{2-29}$$

With this we determine that the SNR is

$$SNR = \frac{E_S}{E_N} = \frac{P_S \tau_p}{kT_0F_n} = \frac{P_T G_T G_R \lambda^2 \sigma}{(4\pi)^3 R^4 kT_0 F_n L} \tau_p \text{ joule/joule or } \frac{\text{w-s}}{\text{w-s}} \text{ or w/w}. \tag{2-30}$$

We note that Equation (2-27) and Equation (2-30) are the same equation if we let

$$B = 1/\tau_p . \tag{2-31}$$

In fact, Equation (2-31) provides us with the ***definition*** of effective noise bandwidth as the reciprocal of the transmitted pulse width.

2.4 EXAMPLE

At this point it will be instructive to consider a couple of examples. For the examples we consider a monostatic radar with the parameters indicated in *Table 2-1*.

Table 2-1 – Radar Parameters

RADAR PARAMETER	VALUE
Peak Transmit Power @ Power Tube, P_T	1 Mw
Transmit Losses, including feed and antenna, L_t	2 dB
Pulse Width, τ_p	0.4 μs
Antenna Gain, G_T, G_R	38 dB
Operating Frequency, f_c	8 GHz
Receive Losses, including feed and antenna, L_R	3 dB
Noise Figure F_n	8 dB
Other Losses, L_{other}	2 dB

For the first example we wish to compute the SNR on a 6-dBsm target at a range of 60 Km. To perform the computation we need to find the parameters in the radar range

[4] We are tacitly assuming that the pulse envelop is rectangular. It is extremely unusual for a radar to have a transmit pulse whose envelop is not rectangular.

equation (Equation (2-27) or Equation (2-30)) and be sure that they are in consistent units. Most of the parameters are in *Table 2-1*, or can be derived from the parameters of *Table 2-1*. The two remaining parameters are the target range and the target RCS, which are given above. The parameters that we will need to compute are the wavelength, λ, and the total losses. If we use Equation (2-27), which we will, we also need to compute the effective noise bandwidth, B. The appropriate parameters are given in *Table 2-2* in "dB units" and MKS units.

Table 2-2 – Radar Range Equation Parameters

RADAR RANGE EQUATION PARAMETER	VALUE (MKS)	VALUE (dB)
P_T	10^6 w	60 dBw
G_T	6309.6 w/w	38 dB
G_R	6309.6 w/w	38 dB
$\lambda = c/f_c$	0.0375 m	-14.26 dB(m)
σ	3.98 m^2	6 dBsm
R	60×10^3 m	47.78 dB(m)
kT_0	4×10^{-21} w-s	-204 dB(w-s)
$B = 1/\tau_p$	2.5×10^6 Hz	64 dB(Hz)
F_n	6.31 w/w	8 dB
$L = L_t L_r L_{other}$	5.01 w/w	7 dB

If we substitute the MKS values from *Table 2-2* into Equation (2-27) we get

$$\begin{aligned} SNR &= \frac{P_T G_T G_R \lambda^2 \sigma}{(4\pi)^3 R^4 kT_0 B F_n L} \\ &= \frac{(10^6)(6309.6)(6309.6)\left((0.0375)^2\right)(3.98)}{(4\pi)^3\left((60\times10^3)^4\right)(4\times10^{-21})(2.5\times10^6)(6.31)(5.01)} \\ &= 27.41 \text{ w/w or } 14.38 \text{ dB} \end{aligned} \tag{2-32}$$

As a double check, we compute Equation (2-27) using the dB values. This gives

$$SNR = (P_T + G_T + G_R + 2\lambda + \sigma) - \left(30\log(4\pi) + 4R + kT_0 + B + F_n + L\right) \tag{2-33}$$

where all of the quantities are the "dB units" from *Table 2-2*. Substituting yields

$$\begin{aligned} SNR &= \left(60 + 38 + 38 + 2(-14.26) + 6\right) \\ &\quad - \left(32.98 + 4(47.78) + (-204) + 64 + 8 + 7\right) \\ &= 14.38 \text{ dB or } 27.42 \text{ w/w} \end{aligned} \tag{2-34}$$

which agrees with Equation (2-32) (except for the last digit of the MKS value).

2.5 DETECTION RANGE

One of the important uses of the radar range equation is in the determination of *detection range*, or the maximum range at which a target has a given probability of being detected by the radar. The criterion for detecting a target is that the SNR be above some threshold value. If we consider the above radar range equations, we note that SNR varies inversely with the fourth power of range. This means that if the SNR is a certain value at a given range, it will be greater than that value at shorter ranges. The upshot of this discussion is that we define the detection range as the range at which we achieve a certain SNR. In order to find detection range, we need to solve the radar range equation for R. Doing so by using Equation (2-27) as the starting point yields

$$R=\left(\frac{P_T G_T G_R \lambda^2 \sigma}{(4\pi)^3 (SNR) k T_0 B F_n L}\right)^{1/4} \text{ m}. \tag{2-35}$$

As an example, suppose we want the range at which the SNR on a 6-dBsm target is 13 dB.[5] Using the *Table 2-2* values in Equation (2-35) yields

$$\begin{aligned} R &= \left(\frac{P_T G_T G_R \lambda^2 \sigma}{(4\pi)^3 (SNR) k T_0 B F_n L}\right)^{1/4} \\ &= \left(\frac{(10^6)\left((6309.57)^2\right)\left((0.0375)^2\right)(3.98)}{\left((4\pi)^3\right)(19.95)\left(4\times10^{-21}\right)\left(2.5\times10^6\right)(6.31)(5.01)}\right)^{1/4} \\ &= 64957 \text{ m or } 65 \text{ Km} \end{aligned} \tag{2-36}$$

We interpret this to mean that the target will be detected at a maximum range of 65 Km. Or, that the target will be detected for all ranges of 65 Km or less.

[5] The value of 13 dB is a standard detection threshold. Later, we will show that a SNR threshold of 13 dB yields a detection probability of 0.5 on an aircraft type of target.