2.5 SEARCH RADAR RANGE EQUATION

We now want to study an extension to the radar range equation that is used to analyze and design search radars. Its most common use is in gross sizing of search radars in terms of power and physical size. In fact, the measure of performance that is usually used to characterize these types of radars is power-aperture product, $P_A A_e$, which is the product of the average power times the effective antenna area of the radar.

To start, we assume that the radar must search an angular region, or sector, which we denote by $\Omega$. $\Omega$ has the units of rad$^2$, or steradians. If the azimuth and elevation extents of the sector to be searched is small,

$$\Omega = \Delta\beta \Delta\alpha \text{ steradians}$$  \hspace{1cm} (2-37)

where $\Delta\beta$ and $\Delta\alpha$ are, respectively, the azimuth and elevation extents of the search sector, in radians. The implication of Equation (2-37) is that the search sector is a rectangle in azimuth and elevation space. In fact, the area can be any shape and represented in any appropriate coordinate system. One of the more common search sectors is section of the surface of a sphere bounded by some elevation and azimuth extents. An example of such a surface is shown in Figure 2-5. In this figure the azimuth extent is $\Delta\beta$ and the elevation extent goes from $\alpha_1$ to $\alpha_2$. As shown in Appendix 1, the angular area of this search sector is

$$\Omega = 2\Delta\beta \cos \alpha_0 \sin \left(\Delta\alpha/2\right) \text{ steradians}$$  \hspace{1cm} (2-38)

where $\alpha_0 = (\alpha_2 + \alpha_1)/2$ and $\Delta\alpha = \alpha_2 - \alpha_1$ and all angles are in radians.

Earlier it was shown that the area of the beam on the surface of a sphere of radius $R$ could be written as

$$A_{\text{beam}} = K_A R^2 \theta_A \theta_B \text{ m}^2.$$ \hspace{1cm} (2-39)

Dividing by $R^2$ results in an angular beam area of

$$\Omega_{\text{beam}} = K_A \theta_A \theta_B \text{ steradians}.$$ \hspace{1cm} (2-40)

With this, the number of beams required to cover the search sector is

$$n = \frac{\Omega}{\Omega_{\text{beam}}} = \frac{\Omega}{K_A \theta_A \theta_B}.$$ \hspace{1cm} (2-41)

Equation (2-41) is ideal in that it essentially assumes a rectangular search sector and rectangular beams. In practice, the number of beams required to fill a search sector is given by

$$n = K_{\text{pack}} \frac{\Omega}{\Omega_{\text{beam}}}$$ \hspace{1cm} (2-42)

where $K_{\text{pack}}$ is a packing factor that accounts for the fact that the beams are actually circular (or elliptic) and that there is some overlap of the beams. A typical value for the packing factor for the case of hexagonal packing with 3-dB beam overlap is $K_{\text{pack}} = 4/3$. 

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Recall that one of the parameters of the $P_A A_e$ is the average power, $P_A$. If a radar has a pulse width of $\tau_p$ and a PRI of $T$ the average power is

$$P_A = P_T \left( \frac{\tau_p}{T} \right) = P_T d$$ (2-43)

where $d$ is defined as the duty cycle of the radar.

One of the requirements imposed on a search radar is that it cover the search sector in $T_S$ seconds. This means that the radar must process signals from $n$ beams in $T_S$ seconds. With this, the time allotted to each beam is

$$T_{beam} = T_S / n.$$ (2-44)

If we allow one PRI per beam we get

$$T = T_{beam} = T_S / n.$$ (2-45)

From Equation (2-41) we have $n = \Omega / K_A \theta_A \theta_B$ which, with Equation (2-45) results in

$$T = T_S K_A \theta_A \theta_B / \Omega.$$ (2-46)

Finally, using $d = \tau_p / T$ results in
\[ d = \frac{\tau_p \Omega}{T_s K_A \theta_A \theta_B}. \] (2-47)

If we substitute Equations (2-47) and (2-43) into the radar range equation we get

\[
SNR = \frac{P_A G_s G_R^2 \lambda^2 \sigma T_p}{(4\pi)^3 R^4 kT_0 F_n L} = \frac{P_A G_s G_R^2 \lambda^2 \sigma T_p}{d (4\pi)^3 R^4 kT_0 F_n L}
\]

\[
= \frac{P_A}{\tau_p \Omega} \frac{G_s G_R^2 \lambda^2 \sigma T_p}{T_s K_A \theta_A \theta_B (4\pi)^3 R^4 kT_0 F_n L}
\]

For the last step we use Equations (2-7) and (2-20) in Equation (2-48) to arrive at the final search radar range equation of

\[
SNR = \frac{P_A A_0 \sigma}{4\pi R^4 kT_0 F_n L \Omega}. \] (2-49)

We note that Equation (2-49) does not contain an explicit dependence upon operating frequency (via \( \lambda \)), antenna gain or pulse width, as does the standard radar range equation. This can be of value in performing first-cut search radar designs in that we needn’t specify a lot of parameters.

It must be emphasized that the search radar range equation leads to a first-cut radar design. At best it provides a starting point for a more detailed design in which the specific parameters not in Equation (2-49) are defined. This will be discussed further in the following example.

### 2.6 Example

As an interesting example we consider a requirement placed on search radars used for ballistic missile defense. Specifically, the SALT I treaty specifies that the power aperture product be limited to \(3 \times 10^6\) w-m\(^2\). Given this limitation, we wish to perform a first cut design of a radar to be used for ballistic missile search.

We start by assuming that we want to search a region of space that extends from 0 to 45 degrees in elevation and 30 degrees in azimuth. Further, we want to cover the search sector in 10 seconds. The targets of interest have a RCS of -10 dBsm and we need to achieve a SNR of 13 dB to declare a detection. Current technology can support a noise figure of 4 dB and total losses of 6 dB. These parameters are summarized in Table 2-3.
Table 2-3 – Search RRE Parameters

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuth Search Extent</td>
<td>30°</td>
</tr>
<tr>
<td>Elevation Search Extent</td>
<td>0° - 45°</td>
</tr>
<tr>
<td>Power Aperture Product</td>
<td>3×10⁹ w-m²</td>
</tr>
<tr>
<td>Search Scan Time, ( T_s )</td>
<td>10 s</td>
</tr>
<tr>
<td>Target RCS, ( \sigma )</td>
<td>-10 dBsm</td>
</tr>
<tr>
<td>Detection SNR</td>
<td>13 dB</td>
</tr>
<tr>
<td>Noise Figure, ( F_n )</td>
<td>4 dB</td>
</tr>
<tr>
<td>Total Losses, ( L )</td>
<td>6 dB</td>
</tr>
</tbody>
</table>

We want to determine the detection range of the radar. Solving Equation (2-49) for \( R \) results in

\[
R = \left( \frac{P_A A \sigma}{4\pi (SNR) k T_0 F_n L} \right)^{1/4} T_s .
\] (2-50)

All of the parameters except \( \Omega \) have been specified. We can compute \( \Omega \) from Equation (2-38) as

\[
\Omega = 2\Delta \beta \cos \alpha_0 \sin (\Delta \alpha / 2) = 2 \left( \frac{\pi}{6} \right) \cos (\pi / 8) \sin (\pi / 8) = 0.118\pi .
\] (2-51)

With this we find

\[
R = \left( \frac{3\times10^6 \times 0.1}{4\pi (10) \times 4\times10^{-21} \times 10^{0.4} \times 10^{0.6} \times 0.118\pi} \right)^{1/4} = 948 \text{ Km} ,
\] (2-52)

which we hope will be sufficient.

Let’s carry the above further and see if we can establish some more of the characteristics of this radar. We start by placing the requirement that the radar operate unambiguously in range. The means that we need to choose the PRI, \( T \), to satisfy

\[
T > \frac{2R}{c} = \frac{2 \times 9.48 \times 10^3}{3 \times 10^8} = 0.00632 \text{ s} = 6.32 \text{ ms} .
\] (2-53)

We will choose \( T = 6.5 \text{ ms} .\)

If we devote 1 PRI per beam then, over 10 seconds we would need to transmit and receive

\[
n = \frac{T_s}{T} \times n = \frac{10}{0.0065} = 1539 \text{ beams} .
\] (2-54)

If we assume a circular beam we can use Equation (2-41) to calculate the beam width from
\[
\theta_A = \theta_B = \sqrt{\frac{\Omega}{K_r n}} = \sqrt{\frac{0.118\pi}{1.65 \times 1539}} = 0.0121 \text{ rad} = 0.7^\circ.
\]  (2-55)

We choose to operate the radar at a frequency of 1 GHz (L-band). This gives a wavelength of
\[
\lambda = \frac{c}{f_c} = \frac{3 \times 10^8}{10^9} = 0.3 \text{ m}.
\]  (2-56)

From Equation (2-12) we have
\[
G_r = G_r = \frac{25,000}{\theta_B^2} = \frac{25,000}{(0.7)^2} = 51020 \text{ w/w} = 47.1 \text{ dB}
\]  (2-57)

We can next use Equation (2-20) to write
\[
A_e = \frac{G_r \lambda^2}{4\pi} = \frac{51020(0.3)^2}{4\pi} = 365.4 \text{ m}^2.
\]  (2-58)

If we assume an efficiency of 60\%, we get a physical area of
\[
A_{\text{physical}} = A_e / 0.6 = 365.4 / 0.6 = 609 \text{ m}^2.
\]  (2-59)

Finally, if we assume a circular aperture we obtain an antenna diameter of
\[
D = \sqrt{\frac{4A_{\text{physical}}}{\pi}} = 27.8 \text{ m}
\]  (2-60)

which is approximately the height of a 7 story building.

As a final calculation, we want the peak power of the radar. We assume that we want a range resolution of 150 m, which translates to a pulse width of 1 µs. With the computed PRI of 6.5 ms we get a duty cycle of
\[
d = \tau_p / T = 10^{-6} / 6.5 \times 10^{-3} = 0.00154%.\]  (2-61)

From Equation (2-58) and the given average power aperture of \(3 \times 10^6 \text{ w-m}^2\) we compute an average power of
\[
P_A = \frac{P_A A_e}{A_e} = \frac{3 \times 10^6}{365.4} = 8210.2 \text{ w}
\]  (2-62)

From this and Equation (2-61) we compute a peak power of
\[
P_T = P_A / d = 8210.2 / 0.000154 = 53.4 \text{ Mw}
\]  (2-63)

which is larger than desired.

One way to reduce the peak power requirement would be to use a longer pulse and employ pulse compression. If we would use a 100 µs pulse, with pulse compression, we would reduce the peak power to 534 Kw, which is a much more reasonable value.
At this point we have a preliminary design for a search radar. In practice this would serve as a starting point for a much more detailed design.