Lasers
PH 645/ OSE 645/ EE 613
Summer 2010 Section 1: T/Th 2:45- 4:45 PM
Engineering Building 240

John D. Williams, Ph.D.
Department of Electrical and Computer Engineering
406 Optics Building - UAHuntsville, Huntsville, AL 35899
Ph. (256) 824-2898 email: williams@eng.uah.edu
Office Hours: Tues/Thurs 2-3PM
Chapter 10: Laser Pumping
Requirements and Techniques

- Excitation or pumping threshold requirements
- Pumping pathways
- Specific excitation parameters for optical pumping
- Specific excitation parameters for particle pumping

Chapter 10 Homework: 7, 9, 10, 14(Bonus)

Cambridge University Press, 2004

All figures presented from this point on were taken directly from (unless otherwise cited): W.T. Silfvast, laser Fundamentals 2nd ed., Cambridge University Press, 2004.
Excitation Pumping Threshold Requirements

• Applied excitation flux is defined as the product of the density of the pumping state and the rate of excitation: $N \Gamma$

• By applying a known excitation rate, one can determine the stead state solution for the upper state density

$$N_u = \frac{N_j \Gamma_{ju}}{\gamma_u} = N_j \Gamma_{ju} \tau_u$$

• Where $N_j$ is the density in the state $j$ from which the excitation occurred. In many cases this would be the ground state previously written as $0$ or $1$

• In other cases such as organic die lasers, $N_j$ is the dye concentration mixed into the solvent

• From our expressions for gain, and assuming that the pumping flux generates significantly more states in $u$ than in the lower state, then one can express the equations for gain as:

$$\sigma_{ul} N_u L = \sigma_{ul} N_j \Gamma_{ju} \tau_u L \approx 12 \pm 5 \quad \text{with no mirrors},$$

$$\sigma_{ul} N_u L = \sigma_{ul} N_j \Gamma_{ju} \tau_u L = \frac{1}{2} \ln\left(\frac{1}{R_1 R_2}\right) \quad \text{with two mirrors}$$
Pumping Pathways: Direct Pumping

- **Direct Pumping**: Upper level excitation from a source (or target) level $j$ which is generally the highly populated ground state, $o$, of the laser species.

- **Optical Pumping**: involves the absorption of pumping light within the gain medium.
  - Common process for solid state and organic dye lasers.
  - We define $B_{ou}$ as a coefficient related to the transition probability $A_{uo}$ of the absorbing transition.

  \[
  \Gamma_{0u} = \frac{I(\nu)}{c} B_{0u} = \frac{I}{c \cdot \Delta \nu} B_{0u}
  \]

- **Particle Pumping**: excited particle collision is used to transfer energy within the gain medium.
  - Common process for gas and semiconductor lasers.
  - $K_{ou}$ is the reaction probability that a collision that a particle $p$ will collide with the laser species and excite it from $o \rightarrow u$

  \[
  \Gamma_{0u} = N_p k_{0u}
  \]
Particle Pumping: excited particle collision is used to transfer energy within the gain medium

- Common process for gas and semiconductor lasers
- $K_{0u}$ is the reaction probability that a collision that a particle $p$ will collide with the laser species and excite it from $o \rightarrow u$

$$\Gamma_{0u} = N_p k_{0u}$$

- If we take into account the average relative velocity between the particle $p$ and the target species, $o$, then one can write:

$$k_{0u} = \bar{v}_{p0} \sigma_{0u}$$

- Yielding an excitation flux rate

$$\Gamma_{0u} N_0 = N_p \bar{v}_{p0} \sigma_{0u} N_0$$

- Or $\Gamma_{0u} = n_e \bar{v}_e \sigma_{0u}^e$ in the case where electron collisions are responsible for the excitation

$\sigma_{0u}^e$ is the velocity-averaged electron excitation cross section
Disadvantages of Direct Pumping

- There may be no efficient direct route from $o \rightarrow u$
  - For optical pumping, the $B_{ou}$ associated with absorption would be too small to produce gain
  - For particle pumping, the excitation cross-section would be too small
- There may be a good route $o \rightarrow u$, but there is a better route from $o \rightarrow l$. I.e., the population density of the lower state in the solution is still too large to allow inversion
- Even though there is a good route for excitation, there may not be a good source of pumping flux available for the $\Delta E$ required.
- Let’s examine the conditions specified in the third disadvantage. Take the minimum pumping power per unit volume as:
  \[ P_{0u} = \Gamma_{0u} N_0 \Delta E_{u0} \]
Pumping Pathways: Indirect Pumping

- **Indirect pumping**: Processes that involve some intermediate level $q$. Three categories exist:
  - **Transfer from below**:
  - **Transfer across**:
  - **Transfer from above**:

- In all three cases the transfer from $q \rightarrow u$ for atoms and electron collisions is
  
  \[ N_p \bar{v}_p \sigma_{qu} N_q \]

- For photon transfer
  
  \[ \frac{I}{c \cdot \Delta \nu} B_{qu} N_q \]
Advantages of Indirect Pumping

• In some cases, the intermediate level, q, will have a lifetime much longer than the upper state lifetime, providing a higher population of q than u. Allowing q to act as a reservoir of population that is energetically near level u upon which it can draw from.

\[ N_q = \Gamma_{0q} N_0 \tau_q \]

• In some cases the pumping probability from o\( \rightarrow \)q is much greater than that from o\( \rightarrow \)u, thereby lowering pumping requirements.

• Transfer from q\( \rightarrow \)u is much more favorable in many circumstances than from q\( \rightarrow \)l or q\( \rightarrow \)o.

• Level q can belong to a different species than u, thus species with q can be exited and store the energy until it donates it to u, allowing one to populate the ratio \( N_u / N_l \) in a very controlled manner.

• The energy level q can have a very broad width, accepting pumping flux over a number of different energies and donating those to a specific narrow energy level u which becomes highly stimulated. This is highly applicable to atoms in which the radiative decay occurs between two high-lying levels such as that of the He-Ne laser which can be broadly pumped to excite He that then collides with Ne to populate a very specific u state.
Advantages of Indirect Pumping

Before transfer

Helium metastable

u

Helium

Neon

After transfer

u

Laser 632.8 nm

Helium

Neon
Indirect Pumping: Transfer from Below

- *Transfer from below:*
- Generally for gas lasers: Ar+, Kr, Xe, and single excitation He-Cd lasers
- The q state accumulates because of its very long lifetime and serves as a storage state
Indirect Pumping: Transfer from Below

- **Transfer from below: Ar+ laser**
Indirect Pumping: Transfer Across

- **Transfer across:**
- Generally for gas lasers: He-Ne, CO₂
- The q and u states must have nearly equal energies

![Diagram of energy levels and transitions](image)

- **Helium**
  - $2^1S_0$
  - $2^3S_1$
- **Neon**
  - Fast decay on strong visible transitions (0.54 - 0.73 μm)
  - 3.39-μm laser
  - 632.8-nm laser
  - 1.15-μm laser

2 metastable storage levels
Indirect Pumping: Transfer Across

- **Transfer across**: He-Ne

![Diagram of energy levels and transitions between He, N₂, and CO₂](image)
Indirect Pumping: Transfer Across

- *Transfer across:* $CO_2$
Indirect Pumping: Transfer Across

- Transfer across: $\text{He}^+\text{-Se}$
Indirect Pumping: Transfer from Above

- **Transfer from above:**
  - In most cases, pump energy can transfer over a wide range of energies
  - q can be in either a single energy state or in a band

- Because q lies above u, the population preferentially decays to u as opposed to lower transfers (remember: probability favors the lower energy state in the transfer level)

- In most cases energy moves from q automatically without any additional stimulus at a very fast rate. Transfer is produced by phonon collisions in a solid and by surrounding electrons or atoms in a gas
- Rates as high as 1012/sec or greater in liquids and solids
- Decay rate is thermalization downward by Boltzmann equilibrium conditions
Indirect Pumping: Transfer from Above

- Transfer from above: Ruby Laser, Nd-YAG
Indirect Pumping: Transfer from Above

- Transfer from above: Semiconductor and Die lasers
Indirect Pumping: Transfer from Above

- Transfer from above: Various Flouride lasers including, KrF

![Diagram showing the process of indirect pumping with various ions and lasers.](image-url)
Specific Optical Pumping Geometries

(a) Direct
Flashlamp → Gain medium

(b) Elliptical cavity
Flashlamp → Gain medium

(c) Double elliptical cavity
Flashlamps → Gain medium

(d) Slab laser amplifier
Reflectors → Flashlamps → Gain medium

(e) Glass slabs at Brewster's angle
Beam → Reflectors → Flashlamps → Gain medium

Increased pumping efficiency

By pumping from both directions at the Brewster angle so not to interfere with the optical pathway of the beam
Specific Optical Pumping Geometries

(f) Line focus

Transverse pumping using lenses to increase flux at the surface of the gain medium for cases where a narrow absorption width helps narrow the emission band. Used in dye lasers

(g) End pumped cavity

Designed for extremely high efficiency pumping of small gas and liquid jets

(h) Semiconductor diode pump laser

(i) Laser plasma pumping (point source)

(j) Laser plasma pumping (line source)
Optical Pumping Requirements

- Conditions for optical pumping are described in terms of how the gain medium can be best designed to take advantage of the pumping flux.
- Considers a simple model consisting of a cylindrical rod of gain medium pumped with light.
- The intensity of light in the medium can be described as:
  \[ I = I_0 e^{-\sigma_0 u N_0 x} = I_0 e^{-\alpha_0 u x} \quad \text{(direct pumping)} \]
  \[ I = I_0 e^{-\sigma_0 q N_0 x} = I_0 e^{-\alpha_0 q x} \quad \text{(indirect pumping)} \]
  \[ g_0 = \sigma_{ul} \Delta N_{ul} \simeq \sigma_{ul} N_u \]

  \[ N_u = N_0 \Gamma_0 u \tau_u = N_0 \frac{B_{0u} I \tau_u}{c \Delta \nu_{0u}} = N_0 \frac{B_{0u} I}{c A_{ul} \Delta \nu_{0u}} \]

  Recall that: \( \tau_u \simeq 1/A_{ul} \)

  \[ B_{0u} = (g_u/g_0)(A_{u0} c^3/8\pi h\eta^3 \nu_{0u}^3) \]

  \[ N_u = \left( \frac{g_u}{g_0} \right) \frac{N_0 A_{u0} c^3 I}{8\pi h\eta^3 \nu_{0u}^3 A_{ul} c \Delta \nu_{u0}} = \left( \frac{g_u}{g_0} \right) \left[ \frac{A_{u0} \lambda_{u0}^2}{4\pi^2 \eta^2 \Delta \nu_{u0}} \right] \frac{\pi N_0 I}{2(h\nu_{0u}) A_{ul}} = \alpha_{0u} \frac{\pi I}{2(h\nu_{0u}) A_{ul}} = \frac{g_0}{\sigma_{ul}} \]
Optical Pumping Requirements

- **Simplified Approximation**
- Assume an absorption length, $l_p$, for pumping flux in the gain medium
- In the case where $\sigma_0 N_0 l_p = 1$

\[
l_p = \frac{1}{\sigma_0 N_0}
\]

\[
\frac{I_p \tau_P}{l_p} = I_p \sigma_0 N_0 \tau_P
\]

\[
\frac{I_p \tau_u}{l_p} = I_p \sigma_0 N_0 \tau_u
\]

\[
N_P = N_u = \frac{I_p \sigma_0 N_0 \tau_P}{h \nu_{0i}} \quad \text{for } \tau_P < \tau_u
\]

\[
N_P = N_u = \frac{I_p \sigma_0 N_0 \tau_u}{h \nu_{0i}} \quad \text{for } \tau_P \geq \tau_u
\]
Transverse Optical Pumping Requirements

- Top figure represents the ideal case
- However, while the elliptical geometry provides optical flux from every direction into the laser rod, it may not do so in all directions at the same optical intensity, thereby producing an asymmetric pumping flux within the laser rod that must be corrected by further improving the pump conditions (double laser flash-pump)

Not generally the case
Diode Optical End Pumping Requirements

- Diode end pumping focuses the light as closely as possible throughout the length of the gain medium. Absorption considerations are key to focusing objectives of the lenses themselves.
Laser Output vs. Optical Pumping

\[ P_{\text{out}} = \sigma_S (P_{\text{in}} - P_{\text{th}}) \]

\[ P_{\text{th}} \propto \frac{1}{\sigma_{ul} \tau_u} \]

\[ \phi = \frac{1}{K} \left( \frac{P_u}{M_u^{\text{th}}} - \frac{P_{u}^{\text{th}}}{M_u^{\text{th}}} \right) \]

\[ \frac{P_{u}^{\text{th}}}{M} = \frac{1}{\tau_u} \]

\[ \phi = \frac{\eta V_C}{\sigma_{ul} c} \left( \frac{P_u}{M_u^{\text{th}}} - \frac{1}{\tau_u} \right) \]

\[ \frac{IV_C}{h \nu_{ul} c} = \frac{\eta V_C}{\sigma_{ul} c} \left( \frac{P_u}{M_u^{\text{th}}} - \frac{1}{\tau_u} \right) \]

\[ I = \eta h \nu_{ul} \left( \frac{R_u}{g_{\text{th}}} - \frac{1}{\sigma_{ul} \tau_u} \right) \]

\[ \sigma_{ul} N_{u}^{\text{th}} \approx \sigma_{ul} \Delta N_{ul}^{\text{th}} = g_{\text{th}} \]

\[ P_{\text{out}} = \sigma_S (P_{\text{in}} - P_{\text{th}}) \]
### Laser Output vs. Optical Pumping

**TABLE 10-2**

<table>
<thead>
<tr>
<th>Laser</th>
<th>$\sigma_{ul}^H$ (m$^2$)</th>
<th>$\tau_u$ (μs)</th>
<th>$\sigma_{ul}\tau_u$ (m$^2$-s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ruby</td>
<td>$2.5 \times 10^{-24}$</td>
<td>3,000</td>
<td>$7.50 \times 10^{-27}$</td>
</tr>
<tr>
<td>Nd:YAG</td>
<td>$2.8 \times 10^{-23}$</td>
<td>230</td>
<td>$6.44 \times 10^{-27}$</td>
</tr>
<tr>
<td>Nd:glass</td>
<td>$3.5 \times 10^{-24}$</td>
<td>315</td>
<td>$1.10 \times 10^{-27}$</td>
</tr>
<tr>
<td>Cr:BeAl$_2$O$_4$</td>
<td>$1.0 \times 10^{-24}$</td>
<td>260</td>
<td>$2.60 \times 10^{-28}$</td>
</tr>
<tr>
<td>Ti:Al$_2$O$_3$</td>
<td>$3.4 \times 10^{-23}$</td>
<td>3.8</td>
<td>$1.29 \times 10^{-28}$</td>
</tr>
<tr>
<td>Li:SAF</td>
<td>$4.8 \times 10^{-24}$</td>
<td>67</td>
<td>$3.22 \times 10^{-28}$</td>
</tr>
<tr>
<td>Li:CAF</td>
<td>$1.3 \times 10^{-24}$</td>
<td>170</td>
<td>$2.21 \times 10^{-28}$</td>
</tr>
<tr>
<td>Erbium</td>
<td>$7.0 \times 10^{-25}$</td>
<td>11,000</td>
<td>$7.70 \times 10^{-27}$</td>
</tr>
<tr>
<td>Nd:YVO$_4$</td>
<td>$3.0 \times 10^{-22}$</td>
<td>100</td>
<td>$3.00 \times 10^{-26}$</td>
</tr>
<tr>
<td>Nd:YLF</td>
<td>$1.2 \times 10^{-23}$</td>
<td>480</td>
<td>$5.76 \times 10^{-27}$</td>
</tr>
<tr>
<td>Yb:YAG</td>
<td>$2.1 \times 10^{-24}$</td>
<td>960</td>
<td>$2.02 \times 10^{-27}$</td>
</tr>
</tbody>
</table>
Electron collisional pumping:

- Occurs when an electric field is applied to a gas medium generating free electrons in a plasma with velocities of $10^6 - 10^7$ m/s.
- These electrons collide with atoms or molecules in their ground state producing transitions to excited states.
- The process is regulated by the fact that momentum transfer due to collisions reduces the energy and thus the electric field of the free electrons.
- Assume an electron travels at an average drift velocity, $v_D$, and an average thermal velocity, $v_e$ of:

$$\bar{v}_D = \mu E = \bar{v}_e = \sqrt{\frac{8kT_e}{m_e \pi}}$$

- Application of the ideal gas law:

$$p = NkT$$

Allows one to evaluate the applied field in a given pressure.

<table>
<thead>
<tr>
<th>Laser species</th>
<th>Laser discharge temperature (K)</th>
<th>Average velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neon (Ne)</td>
<td>400</td>
<td>650</td>
</tr>
<tr>
<td>Argon$^+$ (Ar$^+$)</td>
<td>1,500</td>
<td>890</td>
</tr>
<tr>
<td>Copper (Cu)</td>
<td>1,800</td>
<td>775</td>
</tr>
<tr>
<td>Cadmium (Cd)</td>
<td>600</td>
<td>330</td>
</tr>
<tr>
<td>Selenium X-ray (Se$^{23+}$)</td>
<td>6,000,000</td>
<td>40,000</td>
</tr>
<tr>
<td>Carbon dioxide (CO$_2$)</td>
<td>400</td>
<td>440</td>
</tr>
<tr>
<td>Krypton fluoride (KrF)</td>
<td>400</td>
<td>290</td>
</tr>
</tbody>
</table>

**Average Electron Velocities**

<table>
<thead>
<tr>
<th>Electron temperature in discharge</th>
<th>Average electron velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11,600 K ($kT_e = 1$ eV)</td>
<td>$6.7 \times 10^5$</td>
</tr>
<tr>
<td>23,200 K ($kT_e = 2$ eV)</td>
<td>$9.5 \times 10^5$</td>
</tr>
</tbody>
</table>
Expanded Discussion of Particle Pumping in Plasmas

- Number density of electrons in a plasma

\[ n(v) \, dv = n_e \left( \frac{m_e}{2\pi k T_e} \right)^{3/2} e^{-m_e v^2/2kT_e} 4\pi v^2 \, dv \]

- Find the velocity based on kinetic energy

\[ dv = (2m_e E)^{-1/2} \, dE \]

- Substitute and find the electron energy density

\[ n(E) \, dE = \frac{2n_e}{(\pi)^{1/2}(kT_e)^{3/2}} E^{1/2} e^{-E/kT_e} \, dE \]

- Solve for the puling flux

\[ \Gamma_{0u} = \int_0^\infty n(E) v(E) \sigma_{0u}^e(E) \, dE, \]

\[ \Gamma_{0u} = 2n_e \left( \frac{2}{\pi m_e} \right)^{1/2} \left( \frac{1}{kT_e} \right)^{3/2} \int_0^\infty E e^{-E/kT_e} \sigma_{0u}^e(E) \, dE. \]

- Obtain energy density which can be integrated to obtain the cross section

\[ n(E) v(E) \propto E e^{-E/kT_e} \]
Expanded Discussion of Particle Pumping in Plasmas

\[ n(E)v(E) \propto Ee^{-E/kT_e} \]
Electrical Pumping in Semiconductors

- Current density in a semiconductor is
  \[ j = n_C e v_D \]

- The drift velocity is
  \[ v_D = \frac{d}{\tau_R} \]

- Where \( d \) is the diffusion length of the semiconductor based on the time constant and the diffusion coefficient of electrons in the conduction band
  \[ d \approx (D_C \tau_R)^{1/2} \]