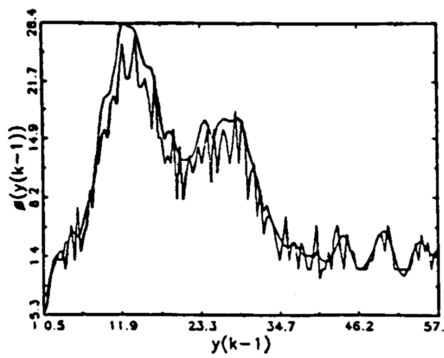

Fig. 10. Noisy nonlinearity and its reconstructed smoothed version by using neural net with MGBF.

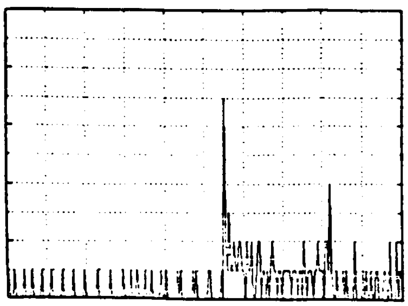

Fig. 11. Histogram of nonlinearity.

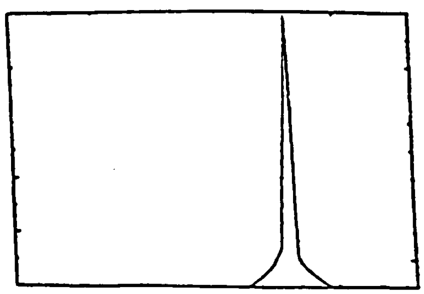

Fig. 12. Histogram of expansion coefficients.

learning rate η to be a function of slow variation

$$\eta = \frac{1}{ln(1 + k)}$$

as suggested by the simulated slow annealing heuristics in optimization, rather than being kept constant, where k here is an iteration number. This modification has substantially improved the network performance as shown in Fig. 13, where the mean-squared error is plotted against time.

VIII. Conclusion

A new representation for the optimal approximation of discrete-time system nonlinearities in terms of a set of Gabor basis functions has been suggested. The application of the neural net approach to solve for appropriate optimal expansion has proved efficient, noise mitigating, and uncertainty tolerant. Main results in this research article include: 1) joint frequency-position domain representation of system nonlinearities; 2) the modified Gabor scheme, and its application to extract a smoothed version of a nonlinearity; 3) investigation of alternative resolution functions, where the simplified scheme has performed as well as the general Gabor scheme; 4) application of the nonuniform sampling scheme to pick the suitable sampling rate, without significant loss of information.

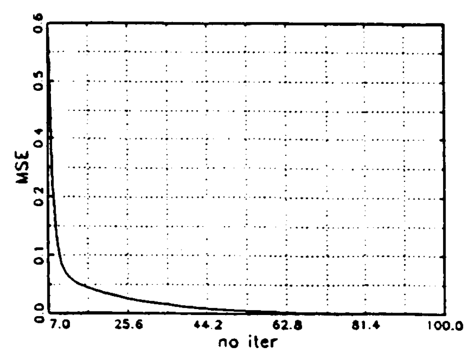

Fig. 13. Mean-squared error versus number of iterations for the neural net with a slowly cooled learning rate.

References

[1] L. Ljung, "Convergence analysis of parametric identification methods," *IEEE Trans. Automat. Contr.*, vol. AC-23, pp. 770–783, Oct. 1978.

[2] J. Hopfield and D. Tank, "Computing with neural circuits: A model," *Science*, vol. 233, pp. 625–633, Aug. 1986.

[3] F. Hoppensteadt, *An Introduction to the Mathematics of Neurons*. Cambridge, U.K.: Cambridge Univ. Press, 1986.

[4] M. Bastiaans, "A sampling theorem for the complex spectrogram and Gabor expansion of a signal into Gaussian elementary signals," *Optical Engineering*, vol. 20, no. 4, pp. 594–598, 1981.

[5] J. Daugman, "Complete discrete 2-D Gabor transforms by neural networks for image analysis and compression," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 36, no. 7, pp. 1169–1179, July 1988.

[6] M. Porat and Y. Y. Zeevi, "The generalized Gabor scheme of image representation in biological and machine vision," *IEEE Trans. Pattern Anal. Mach. Intel.*, vol. 10, pp. 452–468, July 1988.

[7] A. Elramsisi, M. Zohdy, and N. Loh, "Structure and parameter identification of nonlinear discrete-time systems by neural networks," in *Proc. IEEE Conf. Syst., Man, Cybern.*, vol. 3, Nov. 1989, pp. 1089–1104.

[8] A. Elramsisi and M. Zohdy, "An optimal neural net model for image coding in the position-frequency space in the presence of noise," in *Proc. IEEE 1990 Int. Symp. Informat. Theory*, San Diego, CA, Jan. 1990, p. 140.

A Novel Approach to the Design of Unknown Input Observers

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***Abstract*—A novel state estimator design scheme for linear dynamical systems driven by partially unknown inputs is presented. It is assumed**

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that there is absolutely no information available about the unknown inputs, and thus unlike a number of past studies, no *a priori* assumption is made about the nature of these inputs. A simple yet elegant approach for designing a reduced-order unknown input observer (UIO), with pole-placement capability is proposed.

I. Introduction

The problem of estimating the state of a linear time-invariant dynamical system driven by both known and unknown inputs has been the subject of a number of research studies in nearly the past two decades (e.g., see [1]-[8]). The problem is of considerable importance since in practice there are many situations where there are plant disturbances present, or some of the inputs to the system are inaccessible, and therefore a conventional observer which assumes the knowledge of all inputs cannot be used. An observer capable of estimating the state of a linear system with unknown inputs can also be of tremendous use when dealing with the problem of instrument fault detection, isolation, and accommodation (FDIA), since in such systems most actuator failures can be generally modeled as unknown inputs to the system [9], [10].

Basically, there have been two approaches to the problem of designing UIO's. A number of these attempts such as [1], [2], assume some *a priori* information about the unmeasurable inputs. Specifically, Johnson [1] assumes a polynomial approximation to these inputs, and in [2], it is assumed that the unknown inputs can be modeled as the response of a known dynamical system represented by a constant coefficient differential equation. The next category of UIO studies assumes no knowledge of the inaccessible inputs [3]-[8]. Among the earlier works is that of [3], which proposes a simple observer that is capable of reconstructing the entire state of a linear system with the presence of the unknown inputs. However, no systematic guideline for designing such an observer was given in [3]. Since then, several authors have provided various techniques for designing such observers. In [4], the concept of multivariable systems inverse is used, whereas [5] provides a necessary condition for existence of the UIO, and a design based on generalized inverse of matrices. Observers with a similar structure to that of [5] but simpler design techniques were proposed in [6] and [7]. Finally, [8] provides a necessary and sufficient condition for the design of such UIO's.

The UIO design considered in this note falls under the second category of approaches described previously. That is, no knowledge about the nature of the unknown inputs is assumed. In the next section, a very simple and straightforward procedure for designing a reduced-order UIO with pole-placement capability is presented.

II. Design and Structure of the UIO

Consider a linear time-invariant dynamical system represented in the following state-space formulation:

$$\dot{X} = AX + BU + DV \tag{1}$$

$$Y = CX = \begin{bmatrix} 0 & I \end{bmatrix} X \tag{2}$$

where $X \in \mathcal{R}^n$, $U \in \mathcal{R}^q$, $V \in \mathcal{R}^m$, and $y \in \mathcal{R}^p$ are the state, known input, unknown input, and the output of the system, respectively. Notice from (2) that a special structure for the measurement matrix C is assumed. This is not a restrictive assumption since, as long as C is a full-rank matrix, there will always exist a similarity transformation that if performed on a given system will result in the desired output equation [11].

In the following development, a similar condition to that given by [5] and used by others such as [8] is used for designing the UIO. The existence condition in [5] states that for designing a stable observer it is necessary that

$$\operatorname{rank}(CD) = m \qquad \text{with } m \le p. \tag{3}$$

In this study, *we will require that the number of the unknown inputs be less than the number of outputs, that is, $m < p$.*[1]

Using the aforementioned information, the dynamical system described in (1) and (2) can be written in a partitioned form as

$$\dot{X} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} X + \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} U + \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} V \tag{4}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} 0 & I_{(p-m)\times(p-m)} & 0 \\ 0 & 0 & I_{m\times m} \end{bmatrix} X \tag{5}$$

where the matrices $A_1 \in \mathcal{R}^{(n-p)\times n}$, $A_2 \in \mathcal{R}^{(p-m)\times m}$, $A_3 \in \mathcal{R}^{m\times n}$, $B_1 \in \mathcal{R}^{(n-p)\times q}$, $B_2 \in \mathcal{R}^{(p-m)\times q}$, $B_3 \in \mathcal{R}^{m\times q}$, $D_1 \in \mathcal{R}^{(n-p)\times m}$, $D_2 \in \mathcal{R}^{(p-m)\times m} D_3 \in \mathcal{R}^{m\times m}$, and the state vector X is partitioned as

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ y_2 \end{pmatrix}$$

and $x_1 \in \mathcal{R}^{n-p}$ is the vector whose estimate is required.

From the structure of observation matrix C and the necessity condition assumed above, it is easy to show that

$$\operatorname{rank}\begin{pmatrix} D_2 \\ D_3 \end{pmatrix} = m. \tag{6}$$

UIO Design

From (6), without loss of any generality, it can be assumed that D_3 is nonsingular. Therefore, the following matrix operator can be defined

$$T = \begin{pmatrix} I & 0 & -D_1 D_3^{-1} \\ 0 & I & -D_2 D_3^{-1} \\ 0 & 0 & I \end{pmatrix}. \tag{7}$$

Postmultiplying (4) with (7) results in

$$\begin{pmatrix} \dot{x}_1 - D_1 D_3^{-1}\dot{y}_2 \\ \dot{y}_1 - D_2 D_3^{-1}\dot{y}_2 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} A_1 - D_1 D_3^{-1} A_3 \\ A_2 - D_2 D_3^{-1} A_3 \\ A_3 \end{pmatrix} X + \begin{pmatrix} B_1 - D_1 D_3^{-1} B_3 \\ B_2 - D_2 D_3^{-1} B_3 \\ B_3 \end{pmatrix} U + \begin{pmatrix} 0 \\ 0 \\ D_3 \end{pmatrix} V. \tag{8}$$

Notice that the aforementioned algebraic operation is intended to merely eliminate unknown input from the first two rows, and is not a similarity transformation. As a result of performing this operation, it is clear that in (8) the unknown inputs enter only through the third row and the first two rows are independent of any unknown inputs. Thus, defining

$$\bar{A}_i = A_i - D_i D_3^{-1} A_3$$

$$\bar{B}_i = B_i - D_i D_3^{-1} B_3$$

the first two rows of (8) can be written as

$$\dot{x}_1 - D_1 D_3^{-1}\dot{y}_2 = \bar{A}_1 X + \bar{B}_1 U \tag{9}$$

[1] It should be pointed out that this assumption is needed in order to freely assign the eigenspectrum of the observer. In those cases where $m = p$, it is not possible to assign the eigenspectrum of the observer to arbitrary locations, although a stable observer with fixed eigenvalues may be possible [5].

$$\dot{y}_1 - D_2 D_3^{-1} \dot{y}_2 = \bar{A}_2 X + \bar{B}_2 U. \quad (10)$$

Partitioning $\bar{A}_i$ as

$$\bar{A}_i = [\bar{A}_{i1} \quad \bar{A}_{i2} \quad \bar{A}_{i3}]$$

and using it in (9) and (10) will result in

$$\dot{x}_1 = \bar{A}_{11} x_1 + r \quad (11)$$

and

$$z = \bar{A}_{21} x_1 \quad (12)$$

where

$$r = \bar{A}_{12} y_1 + \bar{A}_{13} y_2 + D_1 D_3^{-1} \dot{y}_2 + \bar{B}_1 U$$

and

$$z = \dot{y}_1 - D_2 D_3^{-1} \dot{y}_2 - \bar{A}_{22} y_1 - \bar{A}_{23} y_2 - \bar{B}_2 U.$$

Since the dynamical system represented in (11) and (12) is driven by totally known input r, its state can be estimated by using a conventional Luenberger observer. The dynamics of this observer is given by

$$\dot{\hat{x}}_1 = \bar{A}_{11} \hat{x}_1 + r + M(z - \bar{A}_{21} \hat{x}_1) \quad (13)$$

where M is the observer's gain. Substituting for r and z into (13) results in

$$\dot{\hat{x}}_1 = (\bar{A}_{11} - M\bar{A}_{21})\hat{x}_1 + (\bar{A}_{12} - M\bar{A}_{22}) y_1 + (\bar{A}_{13} - M\bar{A}_{23}) y_2 + (\bar{B}_1 - M\bar{B}_2)U + [(D_1 - MD_2) D_3^{-1} \dot{y}_2 + M\dot{y}_1] \quad (14)$$

Equation (14) contains the derivative of the outputs which is not available for direct measurement. To alleviate this problem a new variable is defined in order to eliminate the need for differentiating the output. To do this let us define a new variable W as follows:

$$W = \hat{x}_1 - [(D_1 - MD_2) D_3^{-1} y_2 + My_1]. \quad (15)$$

Rewriting (14) in terms of the new variable in (15), will result in

$$\dot{W} = FW + EY + LU \quad (16)$$

where

$$H = (\bar{A}_{13} - M\bar{A}_{23}) + (\bar{A}_{11} - M\bar{A}_{21})(D_1 - MD_2) D_3^{-1} \quad (17)$$

$$G = (\bar{A}_{12} - M\bar{A}_{22}) + (\bar{A}_{11} - M\bar{A}_{21}) M \quad (18)$$

$$F = (\bar{A}_{11} - M\bar{A}_{21}) \quad (19)$$

$$L = (\bar{B}_1 - M\bar{B}_2) \quad (20)$$

$$E = [G \quad H]. \quad (21)$$

The following theorem will finally conclude and summarize the design of the UIO.

Theorem: If the pair $\{\bar{A}_{11}, \bar{A}_{21}\}$ is observable, the state of the dynamical system given in (1) can be estimated by using the UIO proposed in (16)–(21). The estimate of the state is given by

$$\hat{X} = \begin{pmatrix} \hat{x}_1 \\ Y \end{pmatrix} = \begin{pmatrix} I \\ 0 \end{pmatrix} W + \begin{pmatrix} N \\ I \end{pmatrix} Y \quad (22)$$

where

$$N = [M \quad (D_1 - MD_2) D_3^{-1}].$$

In addition, all the eigenvalues of F can be placed at any desired location.

The proof of the previous theorem has been implicitly given in the foregoing discussion.

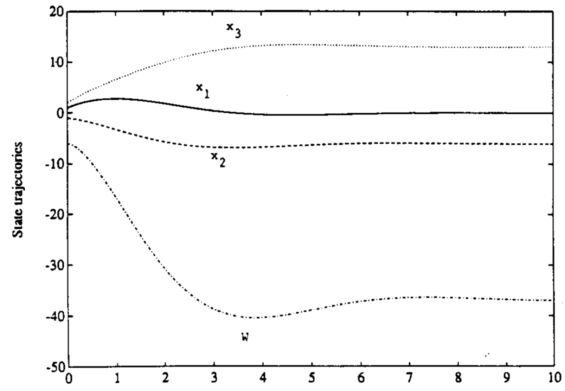

Fig. 1. Response of the UIO and the system.

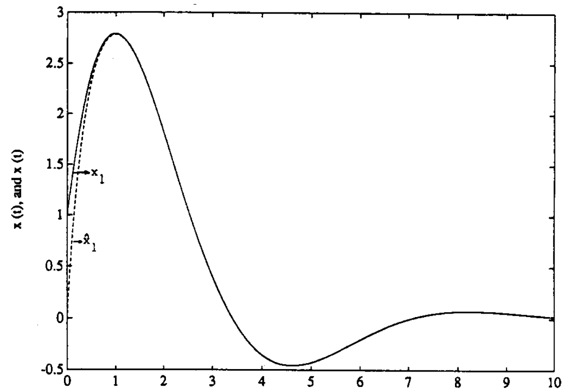

Fig. 2. Actual and estimated state variables.

III. Illustrative Example

In order to illustrate the applicablity of the UIO proposed here, consider the following third-order dynamical system described by

$$\dot{X} = \begin{pmatrix} -1.0 & 1.0 & 0.0 \\ -1.0 & 0.0 & 0.0 \\ 0.0 & -1.0 & -1.0 \end{pmatrix} X + \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} U + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} V$$

$$Y = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} X.$$

Since two of the state variables are directly measurable, a first-order UIO is required to estimate the unmeasurable state variable. Notice also that the observation matrix C is in the desired form described in (2), so no transformation is necessary to get it into the proper form. Preforming the algebraic operation in (8), with the matrix operator T in (7) given as

$$T = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

will result in

$$\begin{pmatrix} \dot{x}_1 - \dot{y}_2 \\ \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} -1 & 2 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & -1 \end{pmatrix} X + \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} U + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} V.$$

The eigenvalues of the open-loop system are all located in the left-hand complex plane and are given by $\{-1, -0.5 \pm j0.866\}$ therefore an observer gain $M = -4$ was selected so that the closed-loop pole of the UIO is placed at -5. Using (16)–(21) the UIO's dynamic was obtained as

$$\dot{W} = -5W + \begin{bmatrix} 22 & -4 \end{bmatrix} Y - U$$

and the state estimate is obtained from

$$\hat{X} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} W + \begin{pmatrix} -4 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} Y.$$

Fig. 1 shows the response of the system along with the observer to a unit step known input and a step input of magnitude 5 for the unknown input with the initial conditions given by $X^T(0) = [1 \ -1 \ 2]$. In Fig. 2, the two (estimated and actual) state trajectories are plotted. From these plots, it is obvious that the UIO is performing as desired.

IV. Conclusion

A new approach for the design of unknown input observers (UIO) capable of estimating the state of a linear dynamical system driven with both known and unknown inputs was presented in this note. By carefully examining the dynamic system involved and simple algebraic manipulations, it was possible to rewrite new equations eliminating the unknown inputs from part of the system and put them into a form where it could be partitioned into two interconnected subsystems, one of which was directly driven by known inputs only. Therefore, this made it possible to use a conventional Luenberger observer with slight modification for the purpose of estimating the state of the system. As a result, it was also possible to state similar necessary and sufficient conditions to that of a conventional observer for existence of a stable estimator and also arbitrary placement of the eigenvalues of the observer. Finally, it is felt that the design and computational complexities involved in designing UIO's is greatly reduced in the proposed approach.

Acknowledgment

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References

[1] C. D. Johnson, "On observers for linear systems with unknown and inaccessible inputs," *Int. J. Contr.*, vol. 21, pp. 825–831, 1975.
[2] J. S. Meditch and G. H. Hostetter, "Observers for systems with unknown and inaccessible inputs," *Int. J. Contr.*, vol. 19, pp. 473–480, 1974.
[3] S. H. Wang, E. J. Davison, and P. Dorato, "Observing the states of systems with unmeasurable disturbances," *IEEE Trans. Automat. Contr.*, vol. AC-20, pp. 716–717, 1975.
[4] N. Kobayashi and T. Nakamizo, "An observer design for linear systems with unknown inputs," *Int. J. Contr.*, vol. 35, pp. 605–619, 1982.
[5] P. Kudva, N. Viswanadham, and A. Ramakrishna, "Observers for linear systems with unknown inputs," *IEEE Trans. Automat. Contr.*, vol. AC-25, pp. 113–115, 1980.
[6] R. J. Miller and R. Mukundan, "On designing reduced order observers for linear time invariant systems subject to unknown inputs," *Int. J. Contr.*, vol. 35, pp. 183–188, 1982.
[7] J. E. Kurek, "The state vector reconstruction for linear systems with unknown inputs," *IEEE Trans. Automat. Contr.*, vol. AC-28, pp. 1120–1122, 1983.
[8] F. Yang and R. W. Wilde, "Observers for linear systems with unknown inputs," *IEEE Trans. Automat. Contr.*, vol. 33, pp. 677–681, 1988.
[9] N. Viswanadham and R. Srichander, "Fault detection using unknown input observers," *Contr. Theory Advanced Technol.*, vol. 3, pp. 91–101, 1987.
[10] Y. Park and J. L. Stein, "Closed loop state and input observer for systems with unknown inputs, *Int. J. Contr.*, vol. 48, pp. 1121–1136, 1988.
[11] C. T. Chen, *Linear System Theory and Design*. New York: HRW, 1984.

Robust Stability and Robust Pole Assignment of Linear Systems with Structured Uncertainty

Yau-Tarng Juang

Abstract—**Sufficient conditions for robust stability of linear time-varying systems are presented. Then the stability criteria are extended to the problem of robust pole assignment of linear time-invariant systems to a specified region. The aforementioned criteria both for robust stability and robust pole-assignment improve some existing results in the literature. Examples are given to demonstrate the new criteria.**

I. Introduction

Stability is a very important and necessary requirement for any workable system. The stability analysis for systems with linear/nonlinear time-varying/time-invariant perturbations has attracted much attention in the literature (e.g., see [1]–[5]). In the linear time-invariant case, it is well known that proper pole assignment will not only ensure system stability but also achieve certain performance. The problem for robust pole assignment has been discussed in the literature (e.g., see [6]–[8]).

In this note, robust stability for linear time-varying uncertain systems is considered in Section II. In Section III, the criteria developed in Section II are applied to the problem of robust pole assignment to a desired region for the linear time-invariant systems. The results improve those presented in [1]–[4] and [7]–[8] for linear systems with structured uncertainty.

II. Robust Stability of Linear Time-Varying Systems

Consider a time-varying matrix $A_c(t)$ which belongs to a convex cone $\mathscr{C}$

$$\mathscr{C} \equiv \left\{ M(t) \mid M(t) = \sum_{k=1}^{r} \alpha_k(t) \mathscr{V}_k(t),\ \mathscr{V}_k(t) \in C^{n \times n},\ \alpha_k(t) \geq 0,\ \sum_{k=1}^{r} \alpha_k(t) \neq 0 \right\}$$

where $\mathscr{V}_k(t)$'s are given matrices.

Theorem 1: The linear time-varying system

$$\dot{x}(t) = A_c(t) x(t) \tag{1}$$

with the closed-loop system matrix $A_c(t) \in \mathscr{C}$ is asymptotically stable if there exists an invertible matrix S such that

$$\mu_2(S\mathscr{V}_k(t)S^{-1}) < 0 \qquad \forall k = 1, 2, \cdots, r \tag{2}$$

where $\mu_2(\cdot)$ denotes the matrix measure [9] corresponding to 2-norm.

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