1. Consider the $R = 1/2$, $K = 4$ code with 

$$G(D) = [1 + D^2 + D^3, 1 + D + D^2 + D^3].$$

(a) Draw the code tree for an information sequence of length $L = 4$.  
(b) Find the code word corresponding to the information sequence $u = (1001)$.  

2. For the binary input, 8-ary output DMC with transition probabilities $P(r_i^j | v_i)$ given by the following table:

| $P(r_i^j | v_i)$ | $r_i = 0_1$ | $0_2$ | $0_3$ | $0_4$ | $1_1$ | $1_2$ | $1_3$ | $1_4$ |
|------------------|----------|------|------|------|------|------|------|------|
| $v_i = 0$       | 0.434    | 0.197| 0.167| 0.111| 0.058| 0.023| 0.008| 0.002|
| $v_i = 1$       | 0.002    | 0.008| 0.023| 0.058| 0.111| 0.167| 0.197| 0.434|

and for the code of Problem 1, find an integer metric table for the Fano metric.  (*Hint:* Scale each metric by an appropriate factor and round to the nearest integer.)

3. Consider the code of Problem 1 and a BSC with $p = 0.045$.  

(a) Find an integer metric table for the Fano metric.  
(b) Decode the received sequence 

$$r = [11, 00, 11, 00, 01, 10, 11]$$

using the stack algorithm.  
(c) Decode the received sequence 

$$r = [11, 10, 00, 01, 10, 01, 00]$$

using the stack algorithm. Compare the final decoded path with the decoded path if the Viterbi algorithm is used.

4. Repeat the example worked in class: $R = 1/3$ code with 

$$G(D) = [1 + D, 1 + D^2, 1 + D + D^2],$$

a metric table given as

| $M(r_i^j | v_i)$ | $r_i = 0$ | $r_i = 1$ |
|------------------|--------|--------|
| $v_i = 0$       | 1      | -5     |
| $v_i = 1$       | -5     | 1      |

and a received sequence of

$$r = [010, 010, 001, 110, 100, 101, 011]$$

(a) Using the stack-bucket algorithm with a bucket quantization interval of 5. Assume that the bucket intervals are $\ldots, +4$ to 0, $-1$ to $-5, -6$ to $-10, \ldots$.  
(b) Using the stack-bucket algorithm with a bucket quantization interval of 9. Assume that the bucket intervals are $\ldots, +8$ to 0, $-1$ to $-9, -10$ to $-18, \ldots$.

5. Repeat Problem 3 for the Fano algorithm with threshold increments of $\Delta = 5$ and $\Delta = 9$. Compare the final decoded path and the number of computations to the results of the examples worked in class and to the results of Problem 3.